VISIT MATH

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Vectors

Topic	Geometry
Learning objectives	Know the characteristics of a vector and how it works. Know how to use vectors and make calculations.
Age group	12-16 years (to be adapted in each country)
Estimated duration	45 mins
Activities	Concrete use of vectors
Related visits	Toulouse.

Previous knowledge required

Definition of a vector

Vector operations

Cartesian coordinates

Step by step: the sequence in the classroom

Step 1: Introducing the topic

Short presentation of the heritage elements in this sequence

In mathematics, a vector is a mathematical object used to represent a quantity that has a magnitude (or size) and a direction. Vectors are commonly used to describe physical quantities such as speed, force, displacement, electric field, and so on. They are essential in many branches of mathematics and physics.

The origin of vectors :

The concept of the vector is the fruit of a long history which began over two thousand years ago. Two families of ideas, initially distinct, are at the origin of its formalisation. One was geometry, dealing with lengths, angles, and the measurement of surfaces and volumes. The other is algebra, which deals with numbers, addition or multiplication, and, more generally, sets with operations.

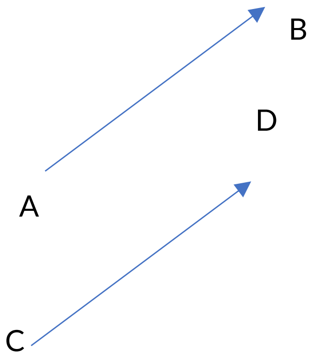
Step 2: Class activities

In class, pupils can tackle a wide range of activities related to mathematical vectors. These activities aim to develop their understanding of basic concepts, their ability to solve problems, and to apply vectors to real-world situations. Here are some examples of activities suitable for this age group:

Introduction to vectors :

- Understand the concept of a vector as a quantity with a length and a direction.
- Represent vectors in a plane using arrows or Cartesian coordinates.

1. Vector concept:



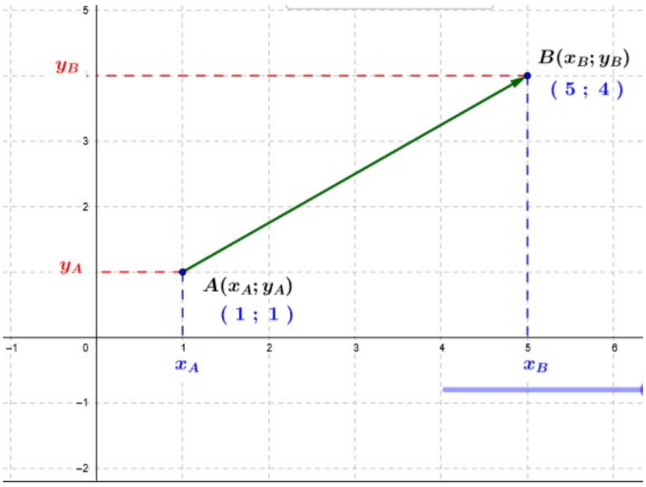
Here, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ have the same magnitude (or size) and the same direction.

A vector can be named with a single letter.

We note $\vec{u}$ this vector and write: $\vec{u} = \overrightarrow{AB} = \overrightarrow{CD}$

Note: The null vector is quite unusual. Unlike other vectors, it has neither direction nor sense! But it is often used in calculations.

2. Cartesian coordinates:



To calculate the coordinates of the vector AB, we use the following formula:

$\overrightarrow{AB} = (x = x_B - x_A ; y = y_B - y_A)$

In this example :

$\overrightarrow{AB}$ = (5 – 1 ; 4 – 1)

$\overrightarrow{AB}$= (4 ; 3)

These coordinates are called the components of the vector, with **x** as the abscissa and **y** as the ordinate. We write: $\overrightarrow{AB} = (x, y)$

Operations :

Pupils can learn to add vectors graphically and analytically.

Visual examples and practical problems can be used. Pupils can draw vectors on paper and use the triangle or parallelogram ruler to add the vectors.

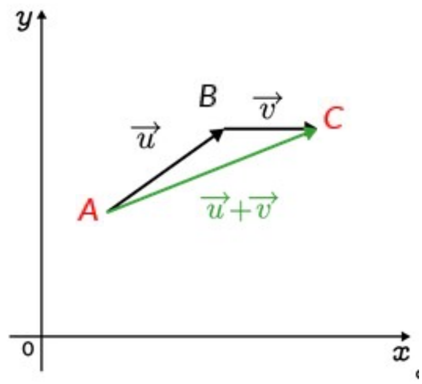
Addition of vectors

- ### Graphical method:

The sum of the vectors $\vec{u}$ and $\vec{v}$ is defined by the following resultant vector $\vec{u}+\vec{v}$.

The triangle method in a cartesian plane produces a resultant vector by forming a triangle, which is the sum of 2 vectors.

Example :



Generally, the addition of two vectors is defined using the Chasles relation:

For all points A, B and C in the plane: $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

To apply the Chasles relationship, the end of the first vector must coincide with the origin of the second. To add two vectors that are not in this configuration, we "carry one of the vectors after the other".

- ### Algebraic method:

Vectors can be added using the algebraic method, by adding together the components of the two vectors.

If you have two vectors $\vec{A}=(A_1, A_2)$ and $\vec{B}=(B_1, B_2)$ then adding $\vec{A}+\vec{B}$ gives a new vector $\vec{C}=(C_1, C_2)$ where each component is obtained by adding the corresponding component of $\vec{B}$ and the corresponding component of $\vec{A}$.

Mathematically, this can be expressed as follows:

$\vec{C}=\vec{A}+\vec{B}=(A_1+B_1, A_2+B_2)=(C_1, C_2)$

Here's a concrete example:

If $\vec{A}$ = (3 ; 2) et $\vec{B}$ = (4 ; 1) so, $\vec{A} + \vec{B}$ = (3 + 4 ; 2 + 1)

$$\vec{C} = (7 ; 3)$$

Also if, $\vec{B} + \vec{B}$ = (4+4 ; 1+1) = (8 ; 2) so $2\vec{B}$ = (2 B_1 ; 2 B_2)

Subtraction of vectors

- **Algebraic method:**

Vectors can also be subtracted component by component.

If you have two vectors $\vec{A}$ = (A_1, A_2) and $\vec{B}$ = (B_1, B_2) then subtracting $\vec{A}-\vec{B}$ gives a new vector $\vec{D}$ = (D_1, D_2) where each component is obtained by subtracting the corresponding component of $\vec{B}$ from the corresponding component of $\vec{A}$.

Mathematically, this can be expressed as follows:

$$\vec{D} = \vec{A}-\vec{B} = (A_1 - B_1, A_2 - B_2) = (D_1, D_2)$$

Here's a concrete example:

Suppose that $\vec{A}$ = (3,1) and $\vec{B}$ = (1,2)

The subtraction $\vec{A}-\vec{B}$ would therefore be :

$\vec{D} = \vec{A}-\vec{B}$ = (3−1,1−2) = (2,−1)

So, $\vec{D}$ =(2,−1)

3. Product of a vector and a real number:

The product of a vector and a real number is a fundamental operation in linear algebra and allows the intensity or direction of a vector to be changed. This operation is often called "scalar multiplication" because it involves multiplying a vector by a scalar (a real number).

Let $\vec{V}$ be a vector and c a real number. The product of the vector $\vec{V}$ by the real number c gives a new vector denoted $c{\cdot}\vec{V}$ or $c\vec{V}$.

Each component of the resultant vector is obtained by multiplying the corresponding component of the original vector by the scalar c.

Mathematically, if $\vec{V} = (V_1, V_2, \ldots, V_n)$ is a vector, then the product $c{\cdot}\vec{V}$ is given by:

$$c{\cdot}\vec{V} = (c{\cdot}V_1, c{\cdot}V_2, \ldots, c{\cdot}V_n)$$

NB: a vector can exist in 3 dimensions as well as in n dimensions!

This means that each component of the vector $\vec{V}$ is multiplied by the scalar c, producing a new vector whose components are c times larger than those of the original vector.

Here's a concrete example:

Let $\vec{A} = (2,-1)$ and $c = 3$. The product $c{\cdot}\vec{A}$ would therefore be:

$$3{\cdot}\vec{A} = (3\text{x}2, 3\text{x}(-1)) = (6,-3)$$

The vector resulting from the product has the same magnitude (or size) and the same direction but is 3 times stronger.

But: $-3{\cdot}\vec{A} = ((-3)\text{x}2,(-3)\text{x}(-1)) = (-6,+3)$

Here, multiplication by a negative real number has changed the direction of the vector.

4. Problems and projects:

Offer exercises and projects that allow pupils to apply the concepts of vectors to real-life situations, such as throwing projectiles, simulating trajectories, etc.
These activities help students develop problem-solving skills, strengthen their understanding of mathematics, and see the practical application of vectors in various areas of science and engineering.
By combining theory with practical activities and real-life examples that show how vectors are used in everyday life and in various areas of mathematics and science, the concept can be made more accessible and interesting to students.

Step 3: Homework and development ideas

Here are some examples of homework and creative activities for learning about vectors that can help reinforce understanding of vectors in maths:

Homework:

1. **Displacement problems:** Give the students a series of displacement problems involving the use of vectors to calculate distances, speeds, and directions. For example, ask them to calculate how long it would take to fly from one city to another, given the wind speed and direction.

Possible general instructions:

- **Initial position:** Give the initial position of the point or object, often represented by an initial vector $\vec{A}$ or the coordinates of point A.
- **Displacement:** Express the displacement as a vector. This can be given explicitly (for example, D ⃗ = (3,-2) for a displacement of 3 units to the right and 2 units down) or implicitly (for example, "move 4 units north").
- **Vector operations:** Use vector operations to calculate the final position. The new position $\vec{B}$ can be found by adding the displacement vector $\vec{D}$ to the initial position $\vec{A}$.
- **Final coordinates:** Provide the final coordinates of the point or object after the move.

Additional questions: Ask additional questions depending on the context of the problem. This could include questions about the total distance traveled, final direction, or other concepts related to travel.

2. **Balancing forces:** Give the pupils situations involving forces applied to an object and ask them to find the resultant. They can also explain whether the object is in equilibrium.
3. **Coordinates and translations:** Ask pupils to solve translation problems using Cartesian coordinates. For example, ask them to describe how a geometric figure moves when subjected to translation vectors.
4. **Modelling projects:** Ask pupils to model a real-life situation using vectors. For example, they could model the movement of a thrown object, a swing, or even the trajectory of a golf ball.
5. **Example:**

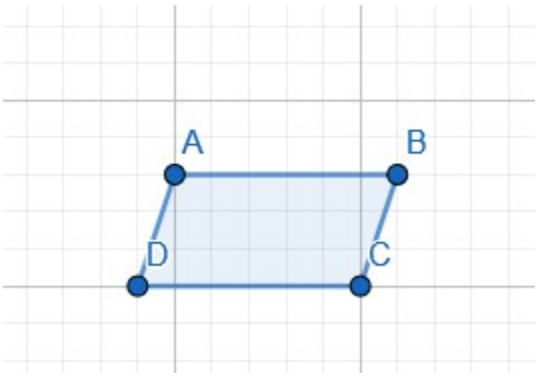
Given a parallelogram ABCD, construct points E, F, G and H such that:

$\overrightarrow{DE} = \overrightarrow{BC}$

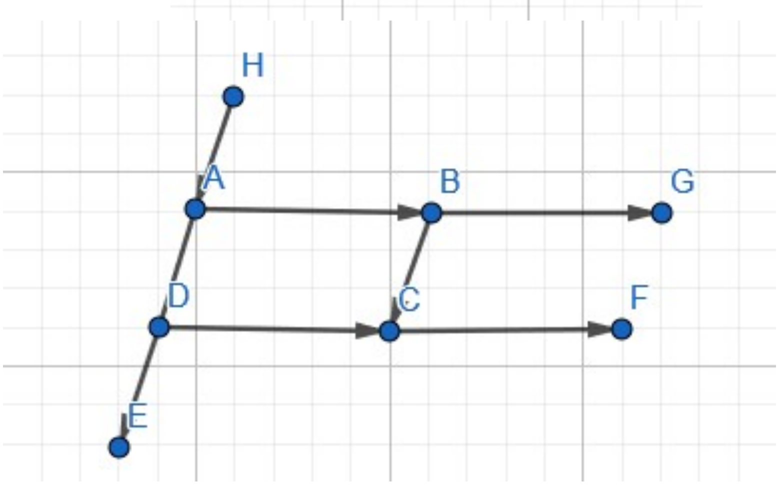
$\overrightarrow{CF} = \overrightarrow{DC}$

$\overrightarrow{BG} = \overrightarrow{AB}$

$\overrightarrow{HA} = \overrightarrow{BC}$



Solution:



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This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Project code: 1-FR01-KA220-SCH-00027771

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