Thales' theorem

Topic	Geometry
Learning Objectives	Using Thales' theorem in different ways
Ages	12-16 years old (to be adapted by each country)
Duration estimate	45 mins
Activities	Discover Thales and applications of his theorem.
Associated visits	Agen, Montauban

Previous knowledge requirements

- Know the basic concepts of geometry, such as points, lines, segments, rays, angles, etc.
- Know the properties of triangles.
- Understand the concepts of ratios and proportions, as well as the ability to solve simple proportional equations.

Step by step: the classroom learning sequence

Step 1: Topic introduction

A little backstory

Thales of Miletus was born around 620 BC in Miletus, Greece. He is considered the first pre-Socratic philosopher, the first of the seven sages of antiquity. He was a mathematician, physicist, astronomer, engineer, and meteorologist. He is the founder of the Ionian School of Natural Philosophy in Miletus.

Aristotle and other ancient philosophers considered Thales to be the first Greek philosopher. Thales refused to go along with previous interpretations of natural phenomena, which until then had only been based on myths, legends, and religious beliefs. Therefore, he managed to approach and explain natural phenomena through scientific logic. He Thales of Miletus is therefore considered the first to have opened the way to scientific research.

Reminders

We will use the following notation conventions:

- AB designating a length between a point A and a point B
- (AB) designating the line of unlimited dimension passing through point A and point B
- [AB] designating the segment, partition of the line (AB), whose ends are point A on the one hand and point B on the other.

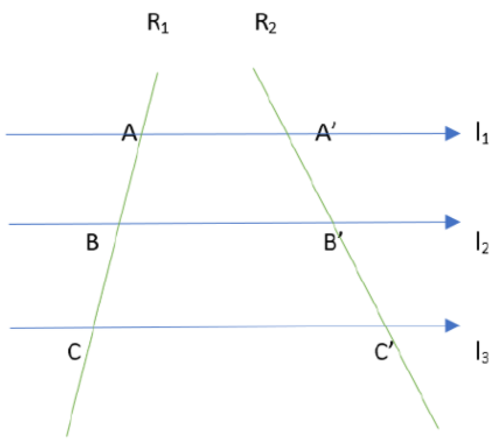
Thales' theorem: a definition

Thales of Miletus was widely known for his theorems and rules among the geometry field. One of them is presented below.

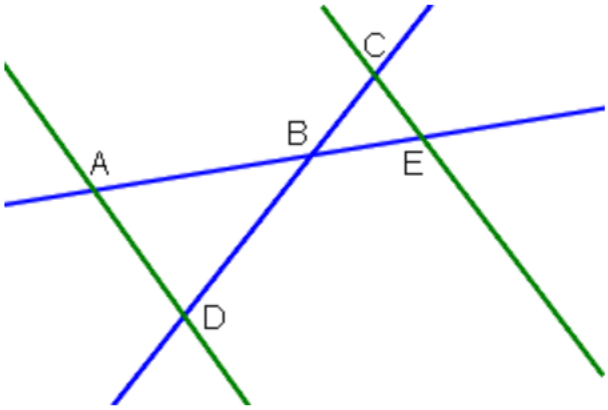
If we consider three straight and parallel lines, l1, l2, and l3, intersecting two other lines, namely R1 and R2, then they produce proportional segments.

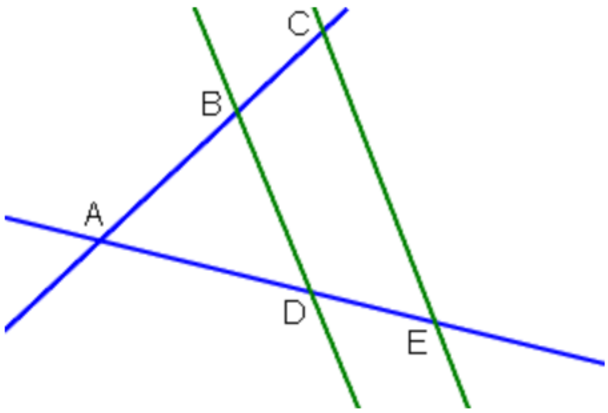
Either if l1 // l2 // l3 and cutting segments through R1 and R2, then:

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$$



Here are two figures for which Thales' theorem could be used:





At least three length measurements must be known in this type of figure for us to use Thales' theorem.

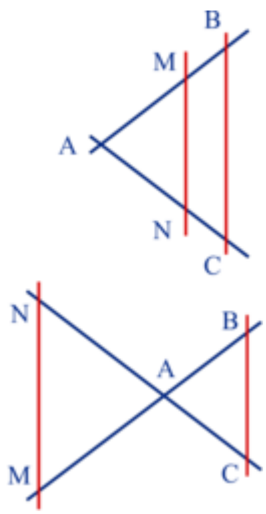
Applying Thales' theorem, therefore, consists of writing equal ratios of lengths in this type of figure.

Step 2: classroom activities

Thales’ theorem: example of application

Thales' theorem states that, in this type of configuration, the sides’ lengths of one triangle are proportional to the associated sides of another triangle. And indeed we have:

$$\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$$



Remarks

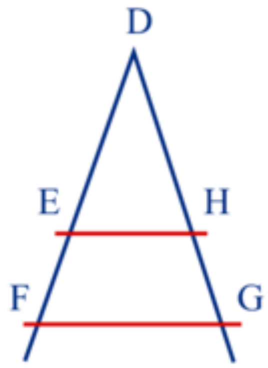
Thales’ theorem cannot be applied if the figure does not **include parallel lines**.

Here: (MN) // (BC).

Upon writing the equality of the three quotients, we put:

- one side of the first triangle as the numerator,
- **the associated side** of the second triangle as the denominator.

Then,



$$\frac{DE}{DF} = \frac{DH}{DG} = \frac{EH}{FG}$$

Example:

In the layout above, suppose we know the lengths of the following segments:

DE=8cm

DF=12cm

DH=4cm

We want to find out the length of DG.

Using Thales' theorem, we can write:

$$\frac{DE}{DF} = \frac{DH}{DG} = \frac{EH}{FG}$$

Let's substitute now with known values:

$$\frac{8}{12} = \frac{4}{DG}$$

By solving this proportional equation, we can calculate the value of DG.

We successfully used Thales' theorem to solve a geometric problem.

DG = $\frac{4*12}{8}$ = 6 cm

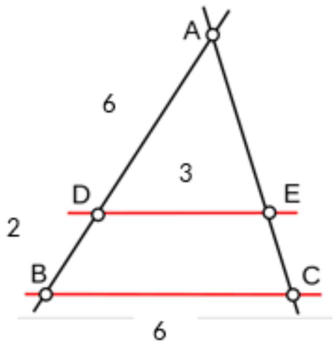
A solution is obtained.

How to verify the parallelism of two lines

In order to check whether two lines are parallel or not, we use the converse of Thales' theorem.

Example:

In order to demonstrate that (BC) and (DE) are parallel, we calculate the two quotients $\frac{AD}{AB}$ and $\frac{DE}{BC}$ separately.



$$\frac{AD}{AB} = \frac{6}{6+2} = \frac{3}{6}$$

$$\frac{DE}{BC} = \frac{3}{6} = \frac{3}{6}$$

$$\frac{AD}{AB} = \frac{DE}{BC}$$

So, according to the converse of Thales' theorem, the lines (DE) and (BC) are parallel.

Step 3: homework ideas and idea development

This theorem can be used to measure many things in your establishment or the pupil environment!

Here is a first exercise that can help develop this idea.

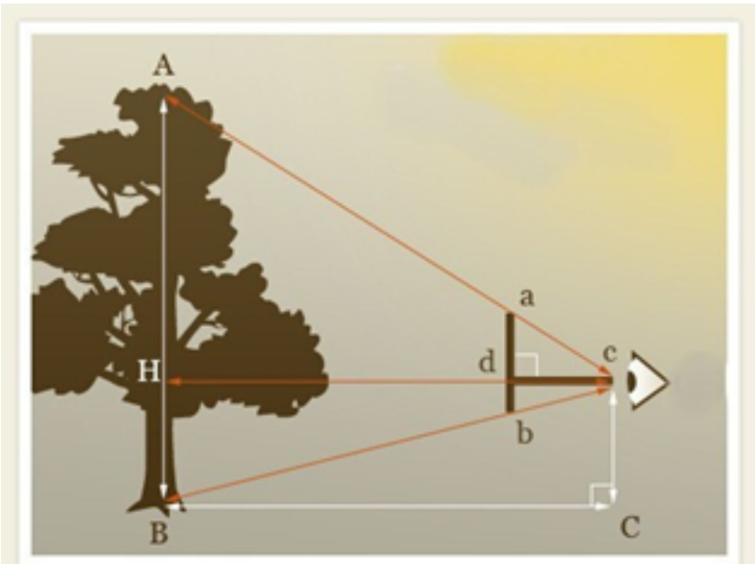
Exercise:

We will use a method called the woodcutter's cross, based on Thales' theorem, to estimate the height of a tree.

This method can also be used to estimate any inaccessible height.

This method requires a T formed by two sticks of the same length.

We consider a tree of height AB at a distance BC from the observer.



Take two sticks of the same size (for example, 20cm) and straight (ab=cd)

Place the first one horizontally (parallel to the ground). Place the second stick perpendicular to the first one.

Then, stand facing the tree at a distance, approximately close to its height.

Then, move forward or backward and slide the vertical stick to align:

- the foot of the tree, the bottom of the vertical stick and your eye *on the same line (cB)*
- the top of the tree, the top of the vertical stick and your eye *on the same line (cA)*

When the two ends of the tree correspond to the ends of the vertical stick, measure the distance separating you from the tree BC.

⇨ **The height of the tree AB is then equal to the distance BC**

Why so?

The lines (ab) and (AB) are parallel due to the recommended positioning, so we can use Thales' theorem.

In the ABc triangle, Thales' rule can be written as this:

$$\frac{ca}{cA} = \frac{ab}{AB}$$

And we can write Thales 's theorem as well for the cHA triangle:

$$\frac{cd}{cH} = \frac{ca}{cA}$$

Hence:

$$\frac{cd}{cH} = \frac{ab}{AB}$$

$Remember, as\ the\ two\ sticks\ are\ the\ same\ lenght,$ **ab = dc**.

And since [cH] is perpendicular to [AB], and the observer is standing vertically, then :

cH = BC.

That means:

$$\frac{ab}{AB} = \frac{ab}{BC}$$

⇨ **AB = BC**

Hence, using Thales' theorem and the so-called "lumber's cross" method, the tree's height is equal to the distance of said tree from the observer.

Now, thanks to Thales' theorem, have fun measuring the height of your school, your city's bridge, a statue...

These videos can complete the sequence:

https://www.youtube.com/watch?v=EOBMCvDMo4M

https://www.youtube.com/watch?v=u_bpbsFZqAA

Co-funded by the European Union

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Project code: 1-FR01-KA220-SCH-00027771

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