VISIT MATH

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Pythagorean theorem

Topic	Geometry
Learning objectives	Use the Pythagorean theorem in different ways.
Age group	12-16 years (to be adapted to each country)
Estimated duration	45 mins
Activities	Discover Pythagoras and the applications of his theorem.
Related visits	MONTAUBAN - AGEN

Previous knowledge required

Basic notions of geometry

Lengths and measurements

Addition and subtraction

Square of a number

Square root

Definition and properties of triangles

Applying geometric formulas

Step by step: the sequence in the classroom

Step 1: Pythagoras and his theorem

Definition of the Pythagorean theorem:

A bit of history

The life and work of Pythagoras are not well known. He was born in the 6th century BC on Samos, an island in the Aegean Sea. After a long initiatory journey, he would have gone into exile in Crotone, south of present-day Italy. There, he founded a school which imposed strict rules of life and took the form of an influential brotherhood. Pythagoreanism is a philosophical, religious, and moral movement, but it is also

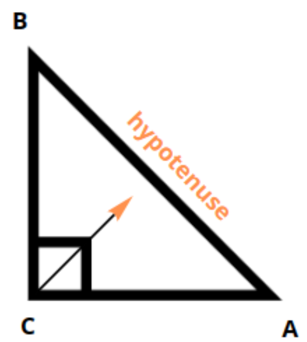
political. It is said that Cylon, one of Pythagoras' opponents, led a revolt against him, spelling the school's end and dispersing the master's followers.
Very early on, Pythagoras became a legendary figure.

Before focusing on defining the theorem, it is essential first to recall the right triangle. Since the use of the Pythagorean theorem only applies to this type of geometric figure.

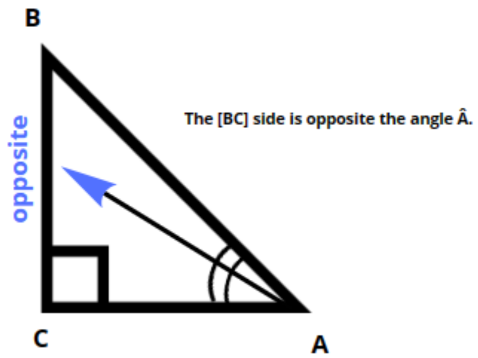
Right triangle:

A right triangle is a triangle that contains a right angle, that is to say, an angle of 90 degrees (90°).
The different sides of a triangle have specific names:

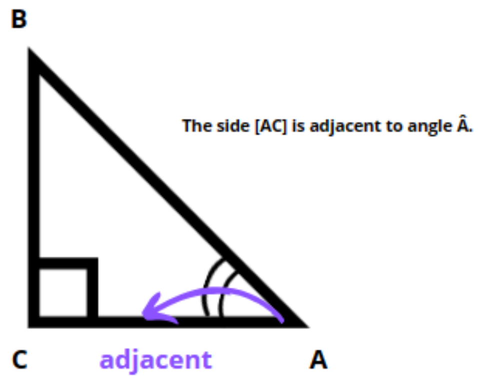
- The hypotenuse is the side opposite the right angle (90°). It is the longest of the three sides.



- The opposite side is one of the sides which constitutes the right angle but is opposite to one of the other two acute angles.



• The adjacent side is one of the sides of an acute angle, not the hypotenuse.



In the case of the Pythagorean Theorem, the main element is the hypotenuse.

The Pythagorean theorem:

The statement of the Pythagorean theorem is as follows:

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

The following formula can summarise it:

$$a^2 + b^2 = c^2$$

where a, b and c represent the lengths of the sides in the right triangle.

This theorem allows you to calculate the length of one side of a right triangle when the lengths of two other sides are known.

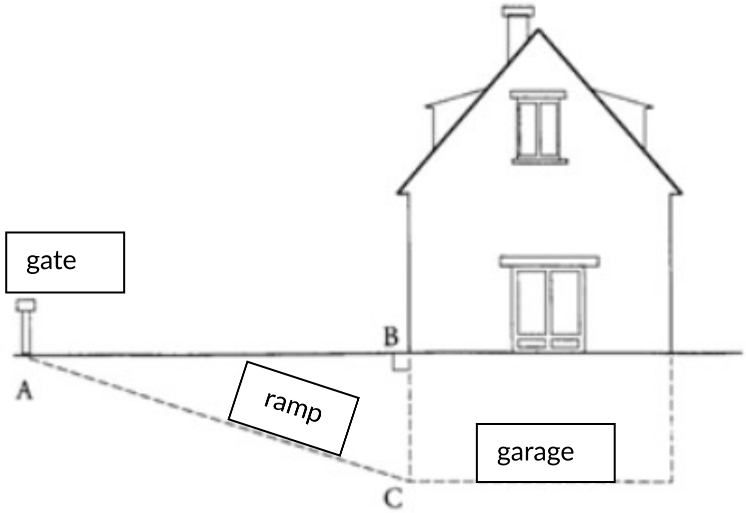
Step 2: Class activities

Example of an exercise to do to understand the relationship between Pythagore and the real-life

To make the exercises more concrete, you can relate them to real-life situations, for example:

Louis lives in a house with a garage 2.25 m below ground. To get there, Louis has to follow a ramp that is 10.25 m long.

What's the distance between the gate and the entrance to his house?



Calculate one side of the right triangle with the Pythagorean theorem formula.

Example

Consider a right triangle with one side measuring 6 cm and the hypotenuse measuring 10 cm.

Can you calculate the length of the last side?

Let “a” be the length of the last side.

Use the formula from the theorem to do the calculation.

$10^2 = a^2 + 6^2$

$100 = a^2 + 36$

$a^2 = 100 - 36$

$a^2 = 64$

$a = 8$

Checking that a triangle is indeed a right triangle.

Since this theorem only applies to the right triangle, we can make a verification.

This is what we call **the reciprocity** of a triangle, that is to say, checking if it is indeed a right triangle.

The converse of the Pythagorean theorem states that if the lengths a, b and c of a triangle verify the equality $a^2 + b^2 = c^2$, then it is indeed a right triangle of which c is the hypotenuse.

Since we know that the hypotenuse is the longest side, we need to add the squares of the two shortest lengths and verify that this sum equals the square of the longest side (the hypotenuse).

Example

Consider a triangle whose lengths are 3, 4 and 5.

Can you prove that it is or is not a right triangle?

To prove that this triangle is a right triangle, we must use the converse of the Pythagorean theorem.

If we apply the formula, we have:

$3^2 + 4^2 = 5^2$

Let's break down this calculation:

$9 + 16 = 5^2$

$25 = 5^2$

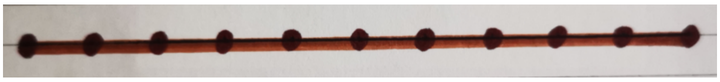
$25 = 25$

Thus, $3^2 + 4^2 = 5^2$ and by the converse of the Pythagorean theorem, it is indeed a right triangle.

Step 3: The rope with 13 knots

The thirteen-knotted rope, also known as the "Druid rope" or "Egyptian rope", was an instrument used in ancient Egypt. It represented an essential tool for builders in the Middle Ages, who used it to transmit construction instructions, understandable by workers unfamiliar with geometry and calculations. Thus, this tool was used to communicate construction plans to workers, thus facilitating the completion of complex architectural projects.

It is a rope with 13 knots, creating 12 regular intervals of equal length. Each knot acts as a mark on the rope, demarcating a space between each centimetre, similar to a tick mark on a ruler. It is important to note that the knot is not a length measurement per se but rather a reference, a sort of graduation on the rope.



This tool seems to allow the teaching of many mathematical concepts.

A stimulating tool for education:

The interest in the “thirteen-knot rope” tool comes from multiple aspects:

- the motivation of teachers and pupils alike, curious and interested in this ancient tool;
- the possibility of linking knowledge in real space and modelling through geometry;
- the implicit differentiation between drawing and figure is fundamental for geometry.

Other developments are possible, in particular, to develop knowledge linked to the circle.

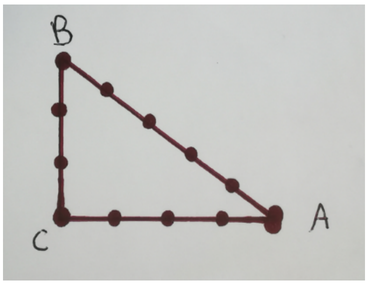
The historical aspect is also essential, allowing pupils to understand the evolutionary path of the construction of knowledge, particularly about the tools available within a population at a given time in its history.

The 13-knot rope is a learning-generating tool whose multiple uses allow pupils to encounter concepts:

- in the field of geometry, it is a powerful tool: plane figure (right triangle, isosceles triangle, equilateral triangle, square, rectangle, rhombus can in particular be materialised by the chord), figure in space, alignment, segment, line, circle, orthogonality, parallelism, polygons, symmetry, middle.

The rope allows you to visualise the square or the diamond by the repetition of 4 identical numbers (3+3+3+3), the rectangles or parallelograms by the repetition of a pair of numbers (1+5+1+5 and 2 +4+2+4), the trapezoids isosceles by the same number in two non-consecutive places (1+3+5+3 or 2+3+2+5), the equilateral triangle (4+4+4) or the isosceles triangle (2+5+5) which has five intervals on two of its sides and two intervals on its base or the right triangle (3+4+5) with its right angle and thus discover the properties allowing us to differentiate the figures between them.

Example for the right triangle:



- in the field of **quantities and measurements**: length, perimeter, area, existence of a polygon of given dimensions (triangular inequality);
- in the **digital domain**: additive decomposition of the number 12, comparison of numbers, etc. The 13-knot rope allows you to add, subtract, multiply, and divide without calculation! For example, folding the rope to have, on each fold, four intervals will enable us to observe that three folds times four intervals = 12 intervals. It is also another excuse to work on fractions.

Step 4: Homework and development ideas

Example of exercises to do to understand the relationship 13-knot rope / Pythagorean theorem

General: How do you prove the Pythagorean theorem with a 13-knot rope?

We use a rope with thirteen knots equidistant, therefore having twelve identical intervals. One person holds the two end knots. Another holds the 4th knot, and one the 7th knot. By stretching the rope, they obtain a perfect right triangle.

Why? Because one side has 3 units of measurement (intervals), the other 4, and the hypotenuse has 5.

$3^2 + 4^2 = 5^2$

The Pythagorean theorem is well verified; the triangle obtained is a right triangle, provided with a right angle.

Examples of instructions:

1. The Egyptians knew how to check that angles were right using a rope with 13 regularly spaced knots. Explain why and how we can check that an angle is right using a 13-knot rope.
2. Imagine a rope with 13 knots stretched between two trees, thus forming a right triangle with the ground. If the chord measures 15 units and one of the sides adjacent to the right corner of the chord measures 9 units, determine the length of the side opposite the right corner using the Pythagorean theorem.
3. An explorer moves through the jungle using a 13-knot rope as a guide. After walking 8 units east, then 5 units north, how far did he walk in a straight line from his starting point?

These exercises allow you to apply the Pythagorean theorem using the 13-knot rope in different contexts. You can solve these problems by identifying the sides of a triangle, applying the Pythagorean theorem, and using the properties of right triangles.

Other activities to ask pupils:

- make a right triangle
- make an isosceles triangle
- make an equilateral triangle
- make a square
- make a rectangle

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