

# Golden ratio

<b>Topic</b>	Mathematics and Geometry
<b>Learning objectives</b>	Understand the concept of the golden ratio and the Fibonacci sequence
<b>Age group</b>	10-16 years (to be adapted in each country)
<b>Estimated duration</b>	2 hours
<b>Activities</b>	Being able to calculate the Golden ratio and Fibonacci sequence
<b>Related visits</b>	Athens

## Previous knowledge required

Understanding planes in Geometry and points

## Step by step: the sequence in the classroom

### Step 1: Introducing the topic

What do a shell and flower petals have in common with Parthenon, Da Vinci's Mona Lisa, and the galaxy? What if someone told you that beauty is simply a mathematical equation?

The answers are the golden ratio and the Fibonacci sequence.



Golden ratio images

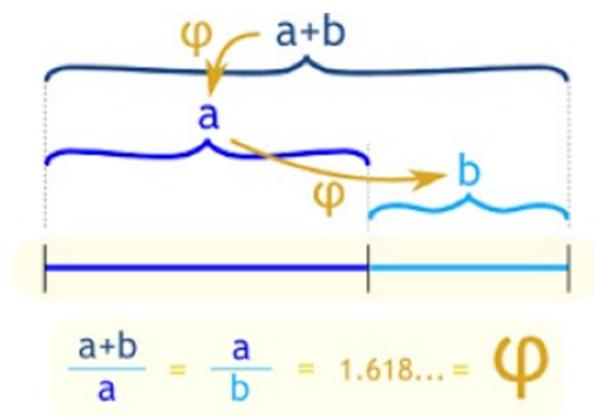
The Greek scholar Pythagoras is credited with discovering the golden ratio. But the origin of this can be traced to Euclid, who mentioned it as the “extreme and mean ratio” in the “Elements”. Then, it was referred to by Luca Pacioli, a contemporary of Leonardo Da Vinci, in "De Divina Proportione" in 1509, by Johannes Kepler around 1600, and by Dan Brown in 2003 in his best-selling novel, “The Da Vinci Code.”

The golden ratio was used by the Egyptians to create their glorious pyramids, by Phidias to design the famed Parthenon, and by artists in the Renaissance as the measurement of all beauty. It is represented by the Greek letter Phi ( $\phi = 1.61803399$ ), known as the Golden Ratio, Golden Number, Golden Proportion, Golden Mean, Golden Section, Divine Proportion, and Divine Section.

The golden ratio is derived from the Fibonacci numbers. Leonardo of Pisa, who was known as Fibonacci, introduced a sequence of numbers to Western civilization in 1202. This sequence, called the Fibonacci sequence, reveals a series of relationships that reflect much of the physical structure of nature.

#### Links between these elements and math topics

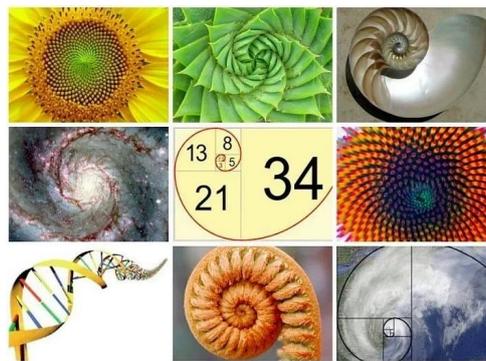
Mathematically, it can be defined as the irrational number  $(1 + \text{Square root of } \sqrt{5})/2$ , often denoted by the Greek letter  $\phi$  or  $\tau$ , which is approximately equal to 1.618. It is the ratio of a line segment cut into two pieces of different lengths, so the ratio of the whole segment to that of the longer segment can be equal to the ratio of the longer segment to the shorter segment.



Golden ratio maths

What is the relationship between the golden ratio and the Fibonacci numbers in the natural world?

The ratio of two consecutive Fibonacci numbers approaches the Golden Ratio. It turns out that Fibonacci numbers show up quite often in nature. Some examples are the pattern of leaves on a stem, the parts of a pineapple, the flowering of an artichoke, the uncurling of a fern, and the arrangement of a pine cone.



Golden ratio examples in various designs

So, why should one study the Golden Ratio?

Do you want to design a face? How to achieve harmony? It's pretty simple—just organize the parts, which are usually dissimilar, into a specific exact ratio so that can they meet and create beauty.

The golden ratio is significant for your connection to nature, as well as the genesis of the universe and the human body. Inspired by its beauty, famous artists have integrated it into their designs and compositions of architectural marvels. For that reason, it helps you to realize the limits of human attention that you can create something that is aesthetically pleasing. If you decide to use the golden ratio as a basis for your art or design, it can help your project look even, balanced, and artistic.

## Step 2: Class activities

### Activity 1



Ancient theatre of Epidauros

Imagine you visit the ancient theatre of Epidauros.

So, if you look at the auditorium, you observe that it has been divided into two, not equal parts.

The one has 34 rows and the other has 21 rows.

Now, check the connection between these numbers and the Fibonacci sequence. What are your findings?

## Activity 2

### Watch this video:

[https://www.pbslearningmedia.org/resource/math\\_nature/fibonacci-sequence/](https://www.pbslearningmedia.org/resource/math_nature/fibonacci-sequence/)

Now let's see some flowers' pictures:



Fibonacci in Nature

Step 1: Create a chart that indicates the type of flower you observed and the number of petals on the flower.

Step 2: Note any observations you make regarding the flower petals.

Is the flower equivalent to numbers from the Fibonacci Sequence?

## Activity 3

Step 1: Search for an online picture of Botticelli's Birth of Venus and print it.

Step 2: Draw a square of size "1" on Venus' body.

Step 3: Place a dot halfway along one side.

Step 4: Draw a line from that point to an opposite corner.

Step 5: Now turn that line so that it runs along the square's side.

Step 6: Then you can extend the square so that it becomes a rectangle.

So, do you believe that Botticelli has used the golden ratio?

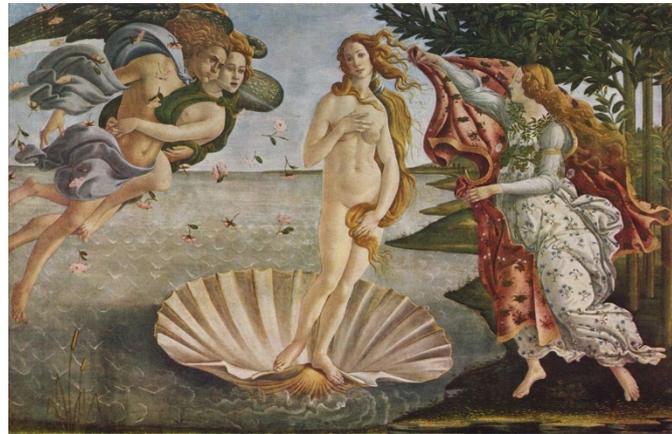
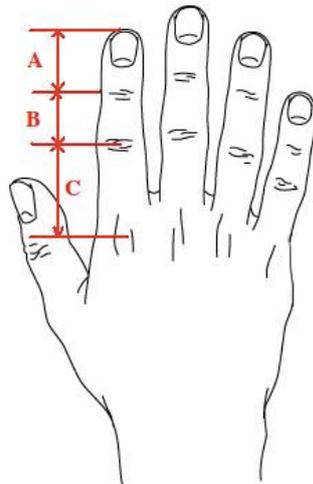


Image Oil painting, Venus, Sandro Botticelli

### Step 3: Homework and development ideas

#### Activity 1



The Golden Ratio and the human body

**Measure the following:**

- Distance from the ground to your belly button
- Distance from your belly button to the top of your head
- Distance from the ground to your knees
- Distances A, B and C
- Length of your hand
- Distance from your wrist to your elbow

Now calculate the following ratios:

1. Distance from the ground to your belly button / Distance from your belly button to the top of your head
2. Distance from the ground to your belly button / Distance from the ground to your knees
3. Distance C / Distance B
4. Distance B / Distance A
5. Distance from your wrist to your elbow / Length of your hand
6. Write all results in a paper
7. Can you see anything special about these ratios?

## Activity 2



Top view of pine cones

Step 1: Go outside and collect pinecones. If you can, gather different kinds and sizes.

Step 2: Look at the widest part of the pinecone and try to see how the seeds form spirals.

Step 3: Use a colored marker to mark all of the spirals going to one direction.

Step 4: Count the spirals. Use a different color marker to mark spirals going to the opposite direction.

Step 5: Count them.

Write out the Fibonacci number sequence on a sheet of paper.

Check the number of spirals you have counted. Are they Fibonacci numbers?  
Do the same with other pinecones!

## References

14 interesting examples of the golden ratio in nature

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