Fractions

Topic	Algebra
Learning objectives	Basic operations around fractions
Age group	10-14 years
Estimated duration	2h
Activities	Use of fractions to calculate shares, prices or times
Related visits	Beaumont de Lomagne, Namur, Paris, Tourcoing/Roubaix

Previous knowledge required

Basic operations, divisions in particular.

Step by step: the sequence in the classroom

Step 1: Introducing the topic

Short presentation of the heritage elements in this sequence

Although it is impossible to know exactly when fractions were first used, several pieces of evidence suggest that the Babylonians already used them more than five thousand years ago. They would use 60 as a denominator, which is believed to be the reason why, for example, there would be 60 minutes in an hour or a full circle would be 360°. The method to calculate them has evolved since, but the idea, however, has stayed: fractions allow to represent a quantity of an item using only natural numbers. Fractions are a representation of a division, and the first people to represent fractions as we know today were the ancient Egyptians. However, their fraction system was mostly designed to use 1 as a numerator, and other numbers on rarer occasions. Still, at this time, proof has been found that the Egyptians knew how to calculate using fractions, such as adding or dividing them. Which is what this lesson will be about!

Links between these elements and math topics

Simple fractions can simply be seen as a visual representation of divisions. Being able to manipulate fractions should help pupils in several ways: this representation means that they do not need to manipulate decimals (which can be confusing sometimes) and that they can be accurate about some measurements (for example, 1/3 can only be correctly written like this). Additionally, the fraction system is used in many parts of our everyday life: telling the time (quarter to eight), cooking instructions (half a cup), asking for a slice of pizza (one-eighth or one-sixth for example) or even when talking about percentages.

Step 2: Class activities

For the teachers: here are some hands-on activities that you can use in class. You may use them and adapt them as you see fit!

What is a fraction?

A fraction is the representation of a division. It is composed of two numbers separated by a horizontal line. The number on top is called the numerator (the number that is divided), and the number on the bottom is the denominator (the number that divides). The denominator cannot be 0.

In the following fraction: $\frac{3}{5}$, 3 is the numerator, and 5 is the denominator.

The biggest number between two fractions that have the same numerator is the one with the smaller denominator. The biggest number between two fractions that have the same denominator is the one with the higher numerator.

Application

In the following examples, which fraction is the bigger one?

$\frac{1}{2}$ or $\frac{1}{3}$?

$\frac{2}{5}$ or $\frac{4}{5}$?

$\frac{3}{8}$ or $\frac{3}{7}$?

$\frac{4}{5}$ or $\frac{4}{9}$?

A natural number can be represented as a fraction with a denominator of 1. You may use this to your advantage when making operations that mix natural numbers and fractions! Likewise, a percentage can be represented as a fraction with a denominator of 100.

Reducing a fraction

The first rule about fractions is that you need to create the smallest fraction possible – it helps with the readability. In order to do so, you need to find a natural number by which to divide both the numerator and denominator. Dividing or multiplying both the numerator and the denominator of a fraction by the same number will create an identical fraction. For example, the following fraction can be divided as such:

$$\frac{4}{6} = \frac{2\times2}{2\times3} = \frac{2}{3}$$

Application

Simplify the following fractions:

$\frac{3}{9} =$

$\frac{3}{6} =$

$\frac{5}{15} =$

$\frac{6}{18} =$

Multiplication

The multiplication is probably the easiest operation when it comes to fractions. When multiplying a fraction by another fraction, multiply the numerators together and the denominators together. When multiplying a fraction by a natural number, multiply the numerator by the number and leave the denominator as it is.

For example: $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$

$\frac{6}{7} \times 3 = \frac{6 \times 3}{7} = \frac{18}{7}$

Application

Now is your turn! Multiply the following fractions, and reduce them when necessary:

$\frac{2}{5} \times \frac{3}{7} =$

$\frac{3}{4} \times \frac{5}{4} =$

$\frac{2}{3} \times \frac{3}{2} =$

Sum

In order to add two fractions, you first need to ensure that both are using the same denominator. In order to do so, you may multiply both fractions by the denominator of the other. Then, add the numerators and do not change the denominator, and reduce the fraction if needed.

For example: $\frac{3}{5}+\frac{5}{2}=\frac{3\times2}{5\times2}+\frac{5\times5}{2\times5}=\frac{6}{10}+\frac{25}{10}=\frac{6+25}{10}=\frac{31}{10}$

Application

Got it? Now is your turn!

$\frac{4}{7}+\frac{3}{4}=$

$\frac{7}{3}+\frac{5}{6}=$

$\frac{5}{3}+4=$

Subtraction

Just like when adding fractions, subtracting them requires you to find a common denominator. Then, subtract both numerators without changing the denominator to find the result.

For example: $\frac{4}{5} - \frac{2}{3} = \frac{4\times3}{5\times3} - \frac{2\times5}{3\times5} = \frac{12}{15} - \frac{10}{15} = \frac{12-10}{15} = \frac{2}{15}$

Application

Let's practice now:

$\frac{7}{8} - \frac{2}{5} =$

$\frac{4}{3} - \frac{2}{6} =$

$\frac{7}{3} - 2 =$

Division

Dividing one fraction by another means multiplying the divided fraction by the reciprocal of the dividing fraction. In other words, you need to switch the numerator and the denominator of the fraction that divides. Since you switch the two numbers, it also means you cannot divide by a fraction that has 0 as a numerator.

For example: $\frac{3}{2} / \frac{5}{4} = \frac{3}{2} \times \frac{4}{5} = \frac{3\times4}{2\times5} = \frac{12}{10} = \frac{6}{5}$

Application

Up to you now:

$\frac{3}{4} / \frac{5}{3} =$

$\frac{5}{7} / \frac{2}{7} =$

$\frac{5}{2} / 3 =$

Step 3: Homework and development ideas

Fractions are part of our everyday life, although we may not notice them at first. The following exercises will show you various cases where knowing about fractions can be useful!

Shopping

Are you ready to do some shopping? Everything is on sale! You have 100€ in your pocket, and you need to buy a t-shirt, a pair of jeans and shoes.

The t-shirt costs 30€ and you can get a 20% discount on it.

The pair of jeans costs 40€ but you can get a 30% discount on it.

The shoes cost 70€ but the price can be reduced by 40%.

Figure 1 Pixabay

Can you afford buying those clothes? How much money do you have left, or how much more money would you need?

The vendor then offers you to purchase a 5€ loyalty card that will grant you an additional 10% discount on the after-sale total. Is it worth it? How much would you pay in total?

Birthday cake

Figure 2 Pixabay

Today is your birthday, and your family is here to celebrate it with you! However, the cake looked so tasty, they all helped themselves before you got home... First, your mother took a small slice: she cut the cake in half and took about an eighth of one half. Then, as your father saw that someone had already eaten some of the cake, he cut the cake into five equal pieces and ate one for himself. Your sister picked the knife again and cut one of your father's slices into two, she ate one of the halves. Finally, your brother came in and ate a third of one of your father's slices.
How much cake is there left? Since there are 5 people in your family, and everyone needs to eat an equal part of the cake, how much can each member still eat?

If you are having trouble solving this problem, you may draw the cake!

The race

Your best friend is participating in a try-out race. They will only qualify if they run faster than 9 km/h! On the day of the race, it took them 20 minutes to complete the 3.5 km race. Did they qualify?

VISIT MATH

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