

Center of Mass

Topic	Geometry
Learning objectives	Understand the concept of Centre of mass
Age group	13-18 years (to be adapted in each country)
Estimated duration	2 hours
Activities	Being able to calculate the Centre of mass using formulas
Related visits	Agrinio

Previous knowledge required

Understating planes in Geometry and points

Step by step: the sequence in the classroom

Step 1: Introducing the topic

Do you know the bird game where the object is to balance itself with its beak on the fingertip? When driving a vehicle, why does the driver's body rotate sharply when approaching a corner?

The answer is "**the Centre of mass**".

The *Centre of mass* is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses.

Look at the past. The Centre of mass has, really, a long and rich history in physics and mathematics. Archimedes, the ancient Greek mathematician, is often credited with laying the foundation for the concept of the Centre of mass. He formulated the principle of the lever, stating that "equal weights at equal distances are in equilibrium." This principle is fundamental to understanding how objects balance around their Centre of mass.

In addition, many notable mathematicians and scientists after them made contributions to the field's research.



1. A bird balancing itself with its beak on a fingertip

Source:

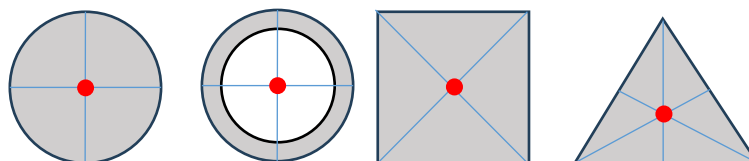
https://en.wikipedia.org/wiki/Centre_of_mass#/media/File:Bird_toy_showing_Centre_of_gravity.jpg

Links between these elements and math topics

Mathematically, it can be defined as the weighted average of the positions of all the particles:

$$x_{cm} = \frac{\sum_{i=1}^N m_i x_i}{M} \quad y_{cm} = \frac{\sum_{i=1}^N m_i y_i}{M} \quad z_{cm} = \frac{\sum_{i=1}^N m_i z_i}{M}$$

The geometry of an object, along with the distribution of mass within it, determines the position of its Centre of mass. If the object has a uniform mass distribution and a symmetrical geometric shape, its Centre of mass is typically located at the geometric Centre.



2. Centre of mass for some simple geometric shapes (red dots)

Do you ask yourself what are the differences between the Centre of gravity and the Centre of mass?

Centre of mass is the point at which the distribution of mass is equal in all directions and does not depend on gravitational field. Centre of gravity is the point at which the distribution of weight is equal in all directions and it does depend on gravitational field.

However, the Centre of gravity is typically located at the same point as the Centre of mass when dealing with objects on or near Earth's surface. It is the point through which the force of gravity acts on the object. Therefore, in our examples, both centres are considered equal, as we deal with objects on the surface of the earth.

So, why should one study the Centre of mass?

Imagine, for example, that you want to construct an arched bridge. How are you going to make sure it is balanced and stable? The Centre of mass helps you to design structures that are safe and functional. For this reason, architects and civil engineers consider the Centre of mass when designing buildings and bridges to ensure they can withstand various loads and environmental conditions.



3. Bridge of Avorani, Agrinio, Greece

Source: <https://www.agrinionews.gr/h-gefyras-tis-avoranis-se-kindyno/>

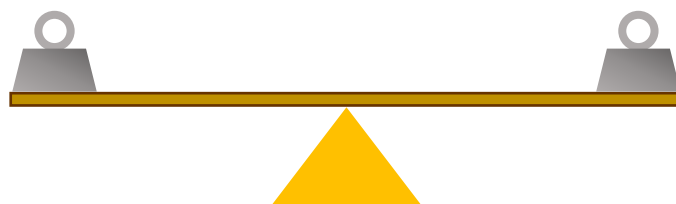
In summary, the Centre of mass is valuable because it provides fundamental insights into the behaviour of objects and systems. It is a critical concept in physics, engineering, and other scientific disciplines, enabling us to analyze and design a wide range of structures, machines, and systems with stability, balance, and efficiency in mind. Engineers use the Centre of mass in structural analysis to assess the stability and integrity of buildings and bridges. It's crucial for ensuring the structure's equilibrium and preventing collapses.

Step 2: Class activities

Seesaw

Imagine you are in a playground and you see a seesaw. How does it work? A seesaw's Centre of mass is its pivot point, as illustrated by the triangle in the image below. With no people on it, the seesaw is perfectly balanced around its Centre of mass.

And even if you put some weights on it and you distribute it evenly, the Centre of mass will be the same, and the seesaw will remain balanced.

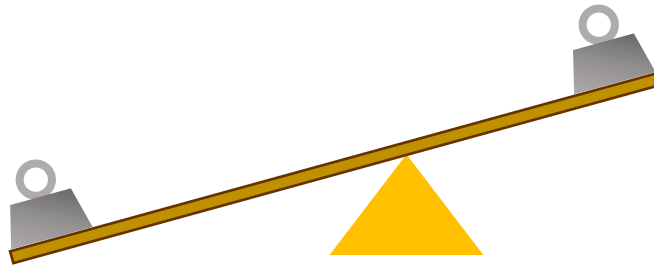


4. Seesaw in balance

Try to build a simple seesaw in your class with some wooden sticks and a small triangle made of hard paper or some children's toy.

If you move the pivot point, then what happens? The seesaw will turn in the direction where there is the greatest distance from the triangle. Why? Because it practically changes the Centre of mass or Centre of gravity, as we see in the second example.

In simple words, in the first seesaw the Centre of balance (or more scientifically the Centre of the mass) is in the middle as the left weight*distance is equal to the right weight*distance. In the second seesaw state the weight*distance is larger than its counterpart, and the Centre of balance changes.

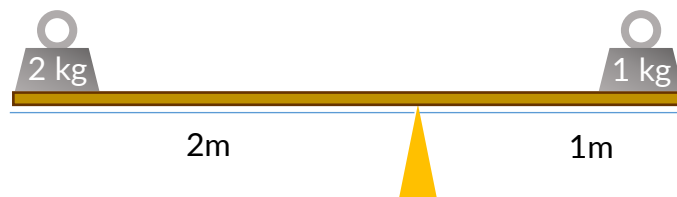


5. Seesaw leaning on its left side

Try to think about what happens if you put heavier weights on one side. What will happen?

Does it balance?

Let's suppose that in the following examples we put a weight of 2 kg 2 meters away from the pivotal point and on the right side a 1 kg 1 meter away from the Centre. Will this arrangement balance? Or is the following figure, correct?



6. Seesaw with different weights and distances for the central point

Since we care only about the x-axis we will calculate the Centre of the mass using the respective part of the equations and referring to meters and not cm. Please note that since the central point is the peak of the triangle, this is considered the point with $x=0$, in the x-axis. So, the object on the left is -2m from it and the object on the right is +1m. We will use the first equation that we show above which concerns the x-axis. Practically, is the sum of the products of the mass of each object times its distance from the central point. Then it is divided by the accumulated mass.

Therefore, the equation becomes:

$$x_m = \frac{\sum_{i=1}^N m_i x_i}{M} = \frac{m_1 * x_1 + m_2 * x_2}{m_1 + m_2} = \frac{2kg * (-2m) + 1kg * 1m}{2kg + 1kg} = \frac{-3}{3} = -1$$

This result shows that the balance point is 1 meter to the left of the triangle. So the system does not balance and the above figure is not correct.

Centre of gravity (mass) and real-life examples

Watch some very interesting and fun videos on how the above concepts are used in real-life experiments.

<https://www.youtube.com/watch?v=qRsJXXb9WNE>

<https://www.youtube.com/watch?v=R8wKV0UQtlo>

Step 3: Homework and development ideas

Estimate the weight of objects

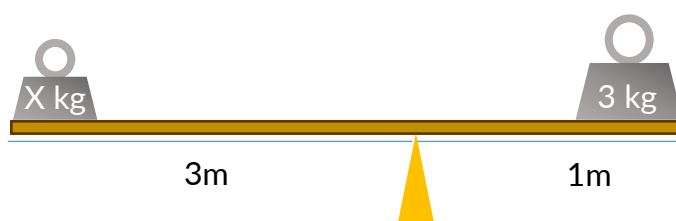
Try to develop a larger seesaw and use it to realize if an object is heavier or lighter than some other object.

And if you know the weight of an object (e.g. 100gr) can you use the seesaw to measure exactly 100gr of sugar?

Search the internet for information on how older scales work.

Find the exact mass of an object

In the following example (figure 8), what is the weight of the object on the left? Take as a fact that the Centre of mass on the x-axis is 0. And the central point is the peak of the triangle.



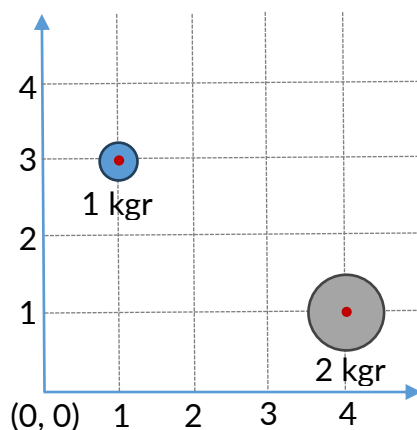
7. Example for calculating the weight

Calculate the Centre of the mass in plane (two-dimensional space)

First, watch the video https://www.youtube.com/watch?v=-nIS_mVjn6A

Then, calculate the centre of the mass of figure 9 in two cases:

- a) With respect to point $(0, 0)$, and
- b) with respect to point $(2, 2)$



9. Example for calculating the Centre of mass

References:

Wikipedia contributors. (2023, November 4). Centre of mass. Wikipedia.

https://en.wikipedia.org/wiki/Centre_of_mass

What is Centre of mass? (article) | Khan Academy. (n.d.). Khan Academy.

<https://www.khanacademy.org/science/physics/linear-momentum/Centre-of-mass/a/what-is-Centre-of-mass>

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