

# Calculus

|                            |   |
|----------------------------|---|
| <b>Topic</b>               | Cartesian coordinate system   |
| <b>Learning objectives</b> | Being able to represent the positions of real objects on the Cartesian plane, interpret graphs and study geometric loci.  |
| <b>Age group</b>           | 14-16 years (to be adapted in each country)   |
| <b>Estimated duration</b>  | 1 h   |
| <b>Activities</b>          | Place the points in the Cartesian plane; determine the position of an object; recognize the characteristics of points that are symmetrical with respect to the axes and the origin. |
| <b>Related visits</b>      | Lucca   |

## Previous knowledge required

Understanding the concepts of oriented line, parallel and perpendicular lines, real numbers, symmetry.

## Step by step: the sequence in the classroom

### Step 1: Introducing the topic

Short presentation of the heritage elements in this sequence



Nicola D'Oresme-  
Wikipedia

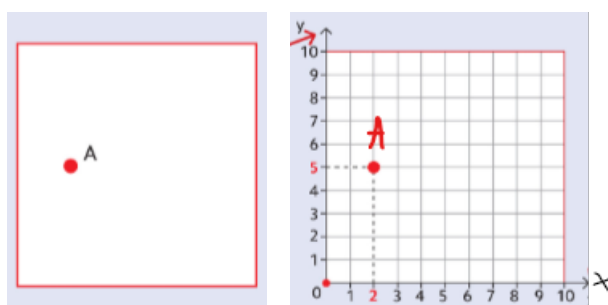
The use of Cartesian coordinates was first introduced by Nicola D'Oresme in the 14th century in Paris, but the term "Cartesian" is attributed to René Descartes (1569 France-1650 Stockholm). Descartes, building on D'Oresme's studies, worked on the fusion of algebra and Euclidean geometry. Legend has it that the young

Descartes, at the age of 23, woke up on 10 November 1619 and declared his intention to transform modern history. His studies led to the development of analytical geometry.

The idea of the coordinate system was developed in 1637 in two of his writings, in the appendix to "Discourse on the Method" and in "How to read the way," simultaneously but independently from Fermat, who, however, didn't publish them. Descartes saw analytical geometry as a "glimmer of light": the coordinate system is like a compass for orientation because it allows us to specify the position of a point or object on a surface.



Descartes-  
Homolaicus.com



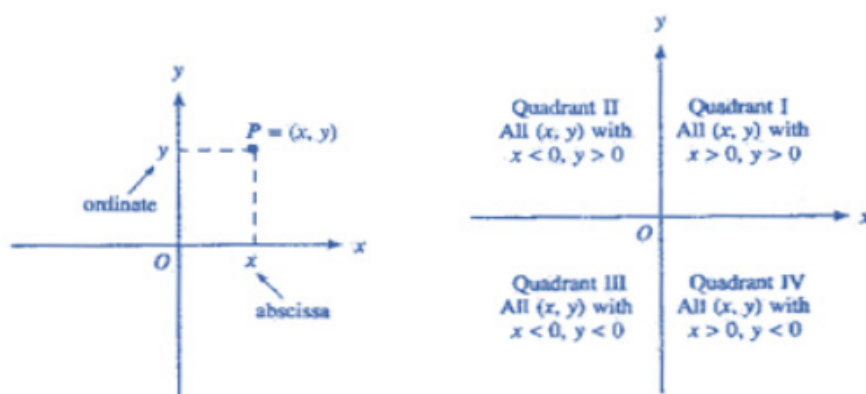
For example: look at this map, where is the treasure?



### Links between these elements and math topics

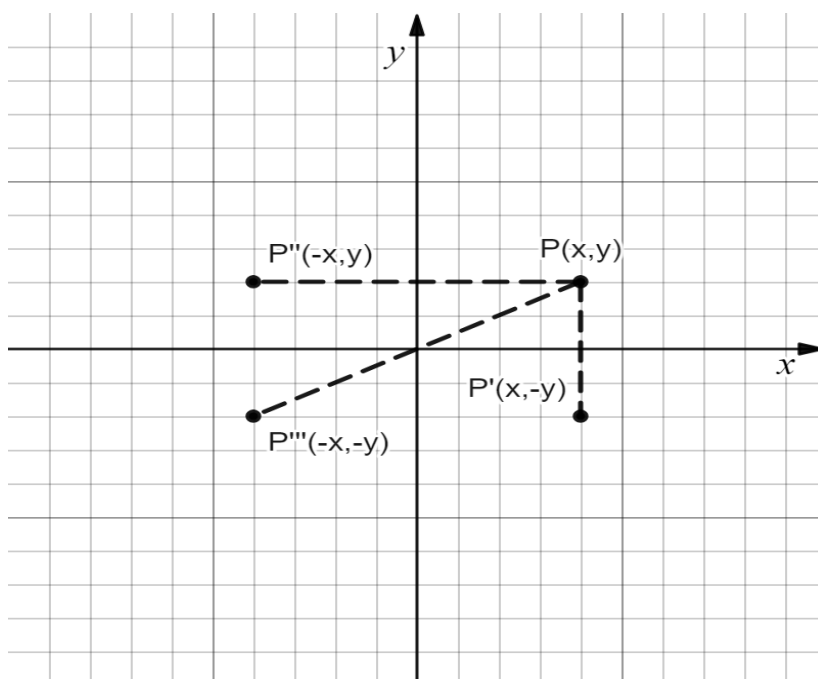
The Cartesian plane is a reference system that allows the representation of mathematical objects in two dimensions. To establish a reference system, it is necessary to draw two perpendicular lines (Cartesian axes) intersecting at a point called the origin. On these lines, we choose a unit of measurement, and each point on

the lines corresponds to a real number. Every point on the plane is identified by an ordered pair of real numbers:  $P(x; y)$ . These are the Cartesian coordinates of point  $P$ , where  $x$  is the abscissa, and  $y$  is the ordinate of  $P$ . The Cartesian plane is divided into four quadrants:



On the Cartesian plane, we can also study symmetries, in particular the symmetry with respect to the  $x$ -axis, the symmetry with respect to the  $y$ -axis and the symmetry with respect to the origin  $O$ .

Take any point  $P(x; y)$  in the first quadrant and draw its symmetric point with respect to the  $x$ -axis. Note that the coordinates of the resulting point  $P'$  are  $(x; -y)$ . Then draw the symmetric point of  $P$  with respect to the  $y$ -axis, obtaining  $P''(-x; y)$ . Finally, determine the symmetric point of  $P$  with respect to the origin:  $P'''(-x; -y)$ .



How can you get it?

## Step 2: Class activities

### Exercise 1

Represent the following points on the Cartesian plane:  $A(2;0)$ ,  $B(-3;2)$ ,  $C(4;-1)$ ,  $D(0;-4)$ , and  $E(-1;-3)$ . What do you notice? What relationship is there between the coordinates of these points and their position on the Cartesian plane?

### Exercise 2

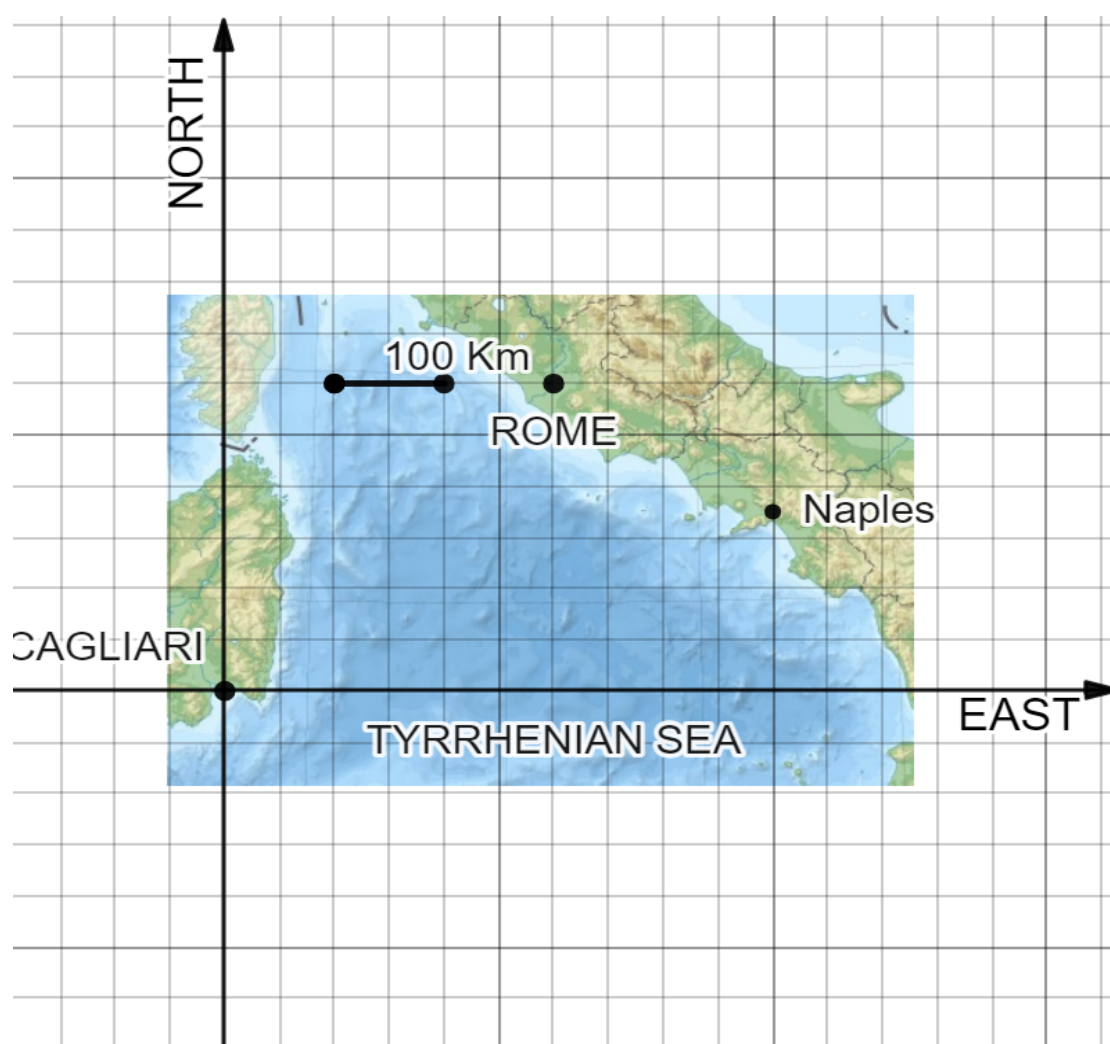
What must be the parameter  $k$  for the point  $P(2k-1;-3)$  to belong to the third quadrant?

### Exercise 3

Let's locate the ship!



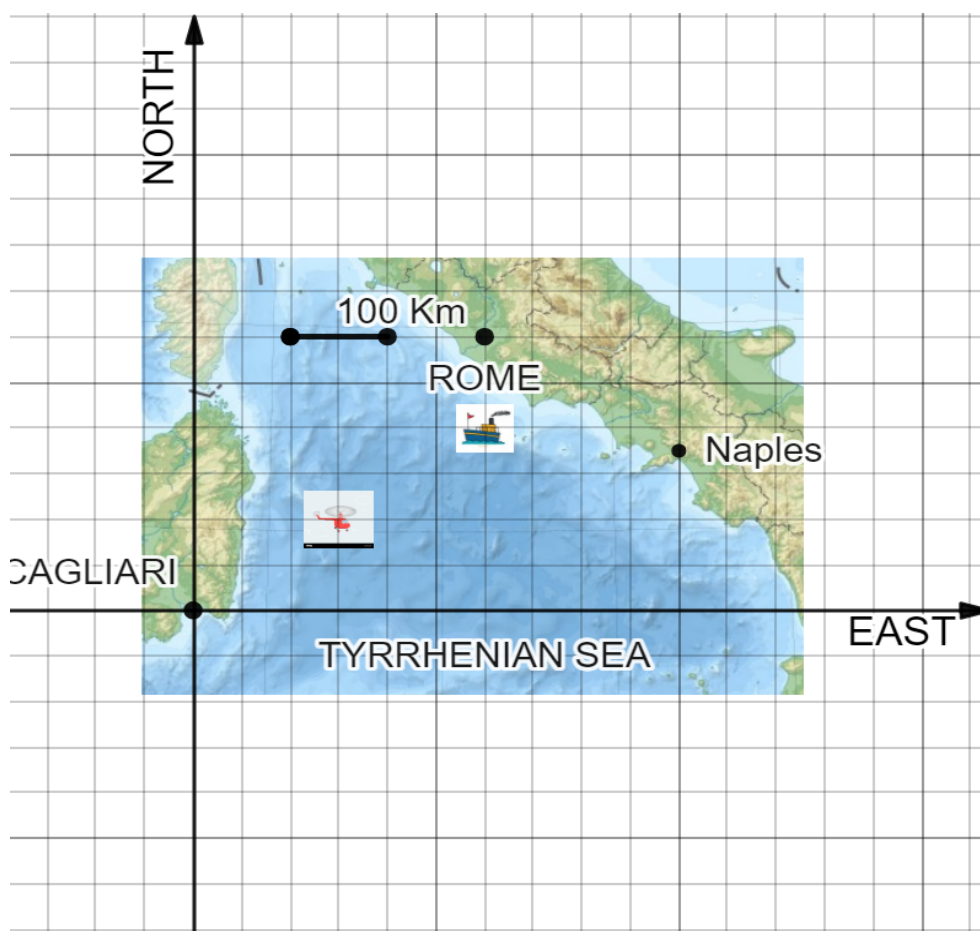
Map of Italy- [matematicafacile.it](http://matematicafacile.it)



A ship is in trouble. A helicopter must go to his rescue. The ship's captain gives its latest position in the Tyrrhenian Sea: 300km east and 200km north of Cagliari. The helicopter pilot has studied the Cartesian plane and he uses it to locate the ship. How does he place the Cartesian axes? Where does the reference system originate? How much can the unit of measurement correspond to?



Map- matematicafacile.it

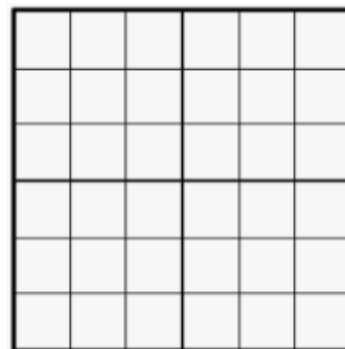


## Exercise 4

Let's play naval battle!

Suppose you have a challenge with a friend of yours who hides a ship 2 squares long in a 6x6 grid.

What is the minimum number of shots you need to fire to be sure of hitting it?



## Step 3: Homework and development ideas

### Exercise 1

Plot the following points on the Cartesian plane and connect them in the order they were given. Do the same for a), b), and c). You will find the answer to the question: "If you lose it, you'll get another one, what is it?"

a) (2;1), (4;1), (4;2), (2;2), (2;3), (4;3)

b) (1;3), (1;1), (-1;1), (-1;3)

c) (-4;2), (-4;3), (-2;3), (-2;2), (-4;2), (-4;1), (-2;1), (-2;2)



## Exercise 2

Look at the map of the city of Chieti. Identify the location of the National Archaeological Museum marked with a red dot. Write the coordinates: (... ; ...)

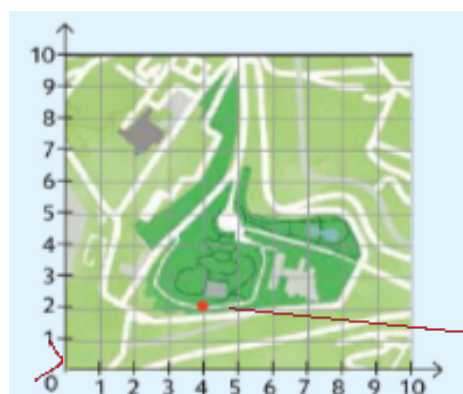


Figure 1 Map of Chieti-Digimparo.it



Figure 2 National Archaeological Museum of Chieti-musA

## Exercise 3

Let  $P(2;1-b)$  and  $Q(a-3;b)$  be, determine  $a$  and  $b$  in such a way that points  $P$  and  $Q$  are symmetric with respect to the  $y$ -axis.

## Material needed for the tour

Paper, pen, ruler.

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