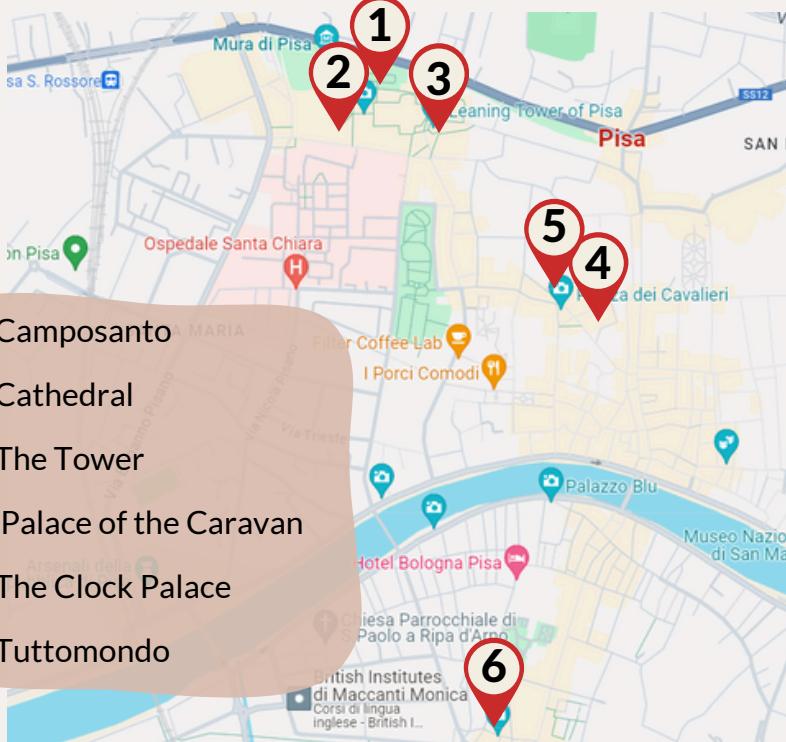


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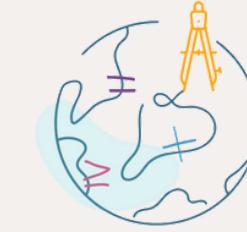


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# Itinerary



- 1 Camposanto
- 2 Cathedral
- 3 The Tower
- 4 Palace of the Caravan
- 5 The Clock Palace
- 6 Tuttomondo



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# VisitMath Tours Pisa



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Project code: 2022-1-FR01-KA220-SCH-000090275



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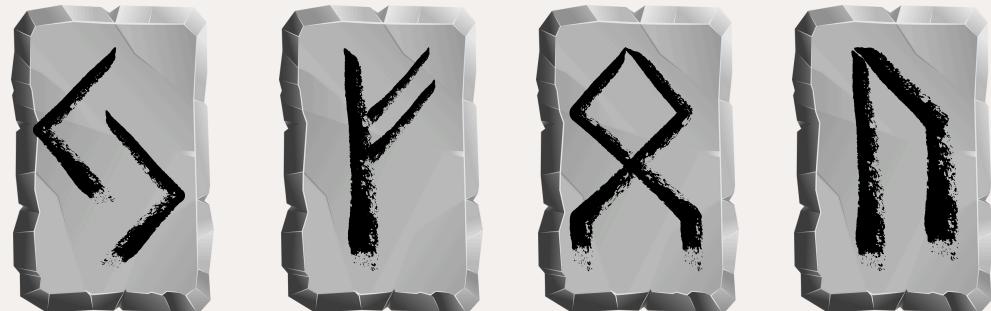
## Step 1: Believe in Mathemagic

You are in the second year of the School of Mathemagic, and your Rune Algebra Professor gives you an important task: recover the extremely rare runes scattered throughout the city of Pisa.



Be careful: the runes are protected by powerful spells that only an exceptional mathemagician can break!

There are four types of runes, each linked to a characteristic of Mathemagic.



Symmetria

Variabilis

Balansius

Sequentia

To break the spell protecting the runes, you will need to solve the exercises posed in clouds like below. If you succeed, you will obtain the runes written on the side!



There is a set of 10 runes, composed by at least 2 runes for each type. How many different combinations are there?



Since you don't have to worry about the order of the runes, you already know that 8 of the 10 runes are determined (2 of each of the 4 types). So you just have to count how many combinations of the remaining 2 runes you can have.



Now you can go to the Camposanto to start the tour

## Step 2: On the sequence

The mission starts from a particular place, the Camposanto of Pisa. You know that some runes are protected by a very powerful magical enchantment that dates back several centuries.



The Cemetery of the Cathedral of Pisa, initiated in 1277 by Giovanni di Simone, stands as the final among the monumental structures adorning the Piazza dei Miracoli. Reverently named the 'Campo Santo' (Holy Field), it was believed that soil from the Holy Land, specifically Golgotha, was brought here by Pisan ships returning from the triumphant Third Crusade.

Crafted from white marble, the exterior showcases 43 blind arches and two entry doors, with an adorned Gothic tabernacle gracing the main entrance. Inside, a captivating cloister with pointed arches unfolds.

The corridors of the cemetery house numerous Roman sarcophagi, serving as the final resting places for notable individuals.

Revealed the mystery, the runes are protected by a spell cast by Fibonacci himself! Will you manage to break it by solving the three challenges?

### Algebra

There are three consecutive terms of the sequence, the sum of the first and the third is equal to 843, the second term is 377.

Calculate the third term.



### Probability

Consider a scenario where a random number is chosen from the first 20 terms of the Fibonacci sequence. Calculate the probability that it is an even number.



### Fibonacci Statue



### Analysis

Use a spreadsheet to calculate the first 15 terms of the Fibonacci sequence and draw a graph of the terms.



Now you can move to Piazza dei Miracoli, looking for runes!

## Step 3: Fight the beast

Some rare Runes are embedded in the wall of the Cathedral. It seems like they have been there for centuries!



The Cathedral, conceived by the brilliance of Busketo (whose remains find their rest in a Roman sarcophagus, a notable part of the facade), began construction in 1063. It was dedicated to the Assumption in commemoration of the triumphant Battle of Palermo, which unfolded between August 13th and 18th of that very year.

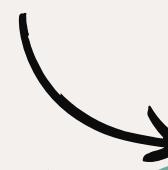
Distinguished by its unique style, the Cathedral is classified as Pisan Romanesque. The facade, a masterpiece by Rainaldo (though many friezes and capitals are credited to Guglielmo and Biduino), features blind arches adorned with lozenges and inlays, capped by four tiers of accessible loggias.

Stretching approximately 100 meters in length and 70 meters in width, the church holds the distinction of being the third-largest in 11th-century Europe, following only St. Peter's and St. Paul's outside the walls of Rome.

There are many small little holes in a piece of marble placed on the northern flank of the Cathedral. Many people stop there and try to count them, but it seems that each time, a different number is obtained. A mystery surrounds the origin of these small holes, known as 'Ungiate' or devil's nails. Legend has it that the devil climbed along the cathedral's flank to the roof to prevent its construction, only to be expelled by divine force, leaving an indelible mark on the stone.



In reality, the creature that left the fingernails is the Avatar of the Uncountable, who has imbued the wall with a powerful enchantment: anyone who directly counts the holes will be wrong! Mathemagic will help you by asking two local guys, Alice and Bob, how many holes they counted.



Alice counts 3 extra nails, Bob counts 2 fewer nails. Twice the number of nails counted by Alice plus half the number of nails counted by Bob equals 380. How many nails does the wall really have?



## Step 4: Hold the tower

What is it? How does it stand? The peculiar building in the square is infused with a powerful mathemagical spell that prevents it from falling. Understanding the workings of this centuries-old spell will unlock some runes embedded in its foundations.



On August 9, 1173, the undertaking of constructing the bell tower for the Cathedral of Santa Maria commenced. A standalone structure, the bell tower reaches a height of approximately 56 meters and boasts a weight of 14,453 tons. Likely overseen by the architect Diotisalvi, who had initiated the construction of the nearby Baptistry, the construction of the tower commenced. However, this initial phase was halted at the level of the third ring due to ground subsidence affecting the foundation on which the tower stands.

The ground's instability, exacerbated by the historical meandering of the Auser River near the tower, led to an unstable foundation, ultimately causing the tower's famous lean. These stability challenges posed significant obstacles, impeding construction progress for an extended period. The resumption of construction in 1275, overseen by Giovanni di Simone and Giovanni Pisano, involved the addition of three extra floors to the original three rings. In an effort to counteract the lean, these supplementary floors were designed to incline in the opposite direction.

The exact inclination of the tower concerning the vertical is  $3.97^\circ$ .



Given that the diameter at the base is 15.484 meters, what is the height difference between the lowest point and the highest point of the base?



Considering that the tower is 56.705 meters tall if the highest point of the last ring is considered, or 55.863 meters if the lowest point of the last ring is considered, how much does the radius of this ring measure?



The mission continues in Piazza dei Cavalieri

## Step 5: The strongest school

In Piazza dei Cavalieri, you arrive in front of the Palace of the Caravan, home to the most powerful school of Mathemagic in Pisa! Will your mathemagic measure up to the prestige of the school?



 The Palace of the Caravan stands as one of the edifices on Piazza dei Cavalieri in Pisa. Originally serving as the headquarters for the Order of the Knights of Saint Stephen, it has functioned as the main seat of the Scuola Normale Superiore in Pisa since 1846. The inception of the Scuola Normale dates back to Napoleon's decree on October 18, 1810, which pertained to 'public education establishments' in Tuscany. This decree established the institution in Pisa as an 'academic boarding school' for university students. Noteworthy achievements within the field of mathematics have flourished at the Scuola Normale, exemplified by recent recognitions such as the Fields Medal awarded to Alessio Figalli for his contributions to the optimal transportation sector.

Entering the school, a teacher challenges you immediately.



Now, instead of an optimal transportation problem, you need to solve an optimization problem regarding transportation!

Will you be able to solve the problem and obtain the runes?



An airline, on a flight with  $n$  passengers, incurs a flight fuel cost of  $40,000 + n^2$  euros and sets the ticket price per passenger at  $400 + 6000/n$  euros. What is the optimal number of passengers it should have to maximize the profit?



When you correctly write the cost function, it forms a parabola, so the maximum is a specific point on it... the vertex!

## Step 6: Clock cypher

While staying in Piazza dei Cavalieri, you notice a peculiar building, imbued with arcane mathematical energies.



The Clock Palace in Piazza dei Cavalieri stands as an ancient medieval structure that served as the residence for the Captain of the People, dating back to at least 1357. Originally owned by the Gualandi family, the palace's construction also encompassed the renowned tower of La Muda or the 'Tower of Hunger.' This tower gained historical significance when, in 1289, Count Ugolino Della Gherardesca met his demise alongside his children and grandchildren within its confines.



Count Ugolino became a prominent figure in Dante's Divine Comedy, specifically in Canto XXXIII. In this section, Dante accuses the traitors of the homeland, and Count Ugolino assumes a central role in the narrative.



An ancient message has been encoded using a polyalphabetic cypher of the Vigenère type, and as the key, it employs the verses from the XXXIII canto of the Divine Comedy that refer to Count Ugolino.

**Tu dei saper ch'i' fui conte Ugolino,  
e questi è l'arcivescovo Ruggieri:  
or ti dirò perché i son tal vicino.**

It seems that deciphering this message could reveal precious arcane runes.



The message found is:  
**'UKUP KBEA GNGMZ CGJGZT'**.  
What could it mean?



Since it is a Vigenère cypher, you have to associate at each letter the corresponding number following the alphabet order, starting with A → 0. So A → 0, B → 1, ..., Z → 25, and then making the sum between the corresponding letters of the message and the key. For example, to calculate the first letter, you pick the first letter of the message that is U, the first of the key that is T, and you make the sum U+T=20+19=39; since it is bigger than 25, you have to subtract 26, so the first letter is 39-26=13=N.



Last you can go to the Church of S. Antonio

## Step 7: Colour magic

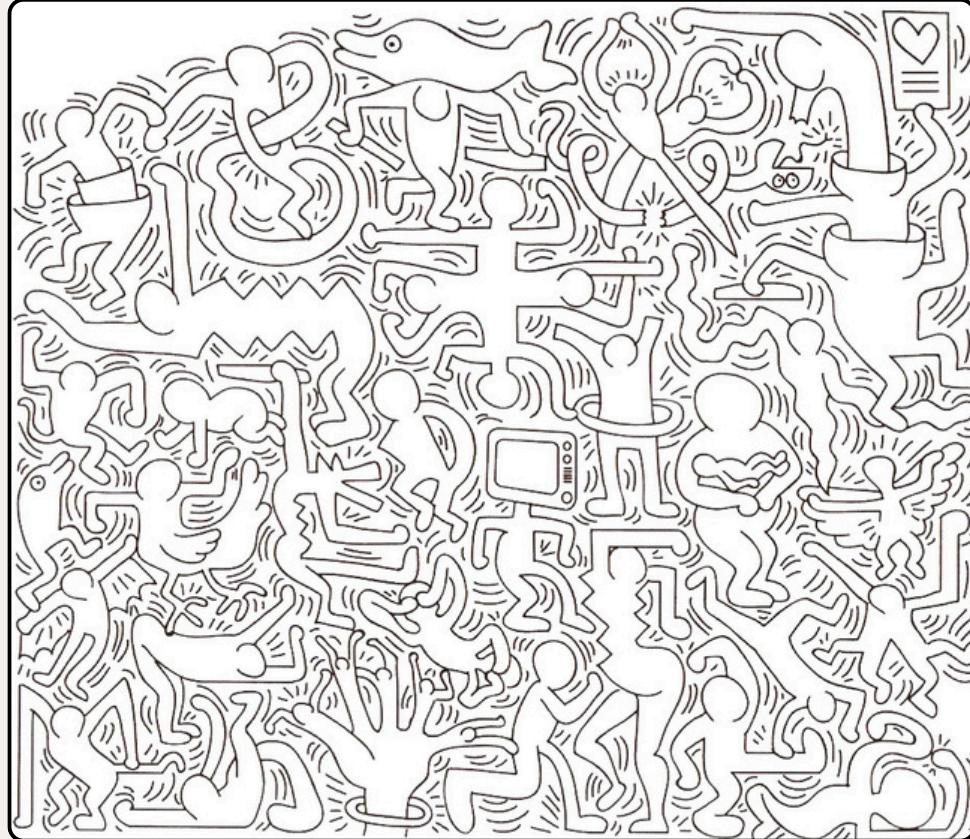
At the Church of S. Antonio, you realize that the last runes are protected by a powerful enchantment of chromatic mathemagic. To break the chromatic bond created by Keith Haring's mural, you must use fewer colours than he did!



A distinctive mural adorns the rear wall of the convent belonging to the 'Servi di Maria' friars at the Church of S. Antonio. In 1989, Keith Haring immortalized his art in Pisa by crafting the mural titled 'Tuttomondo', leaving a lasting imprint on the cityscape.



Below is the mural *Tuttomondo* without colours.



Try to colour Keith Haring's mural with the fewest possible number of colours, so that no region of one colour touches another of the same colour.



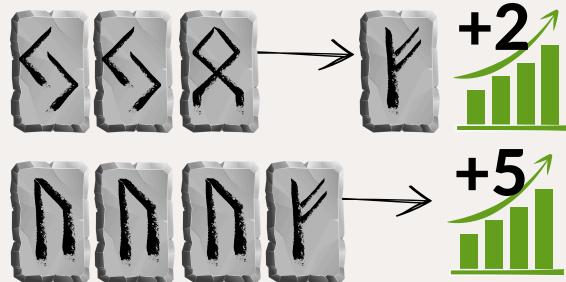
Remember that the four colours theorem states that is always possible doing this with only four colours.

## Step 8: Rune Combo

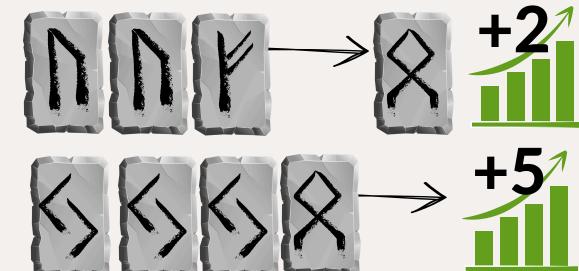
Now you can use the runes obtained in Pisa to face the challenges posed by the Grand Mathemagicians. Choose a challenge, subtract the number of runes it asks for, and receive the reward! You can take on challenges as long as you still have runes. What level  of magic have you achieved?



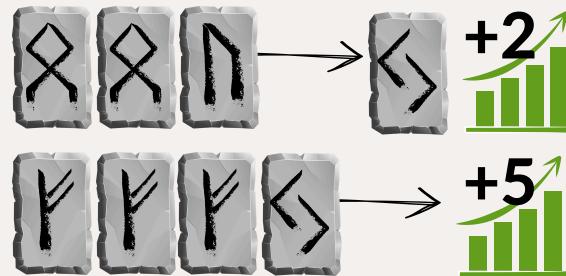
Diophantus



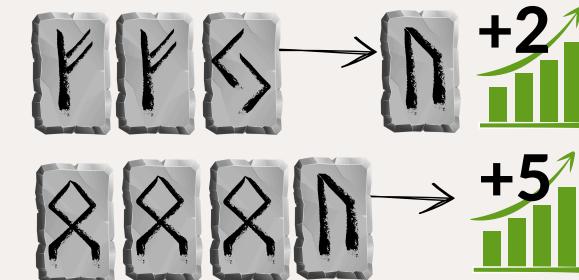
Archimedes



Pythagoras



Thales



Euclid

