VISIT MATH

Co-funded by the European Union

How our cities can help teach mathematics

GUIDE

FERMAT SCIENCE
Une autre idée des maths
5th HIGH SCHOOL
Agrinio · Greece
ISTITUTO STATALE "E. MONTALE" PONTEDERA
LogoPsyCom.
ZS 31
YuzuPulse

Contents

1. Introduction

This guide wishes to shed light on the contribution that European architectural heritage can have in the learning of mathematics by means of an innovative approach. In this context, it is crucial to initially assess the locale and proficiency of European learners as this assessment serves as a foundation for devising targeted strategies to enhance mathematics education.

What the studies say on this subject paints a rather negative picture of the situation. Indeed, according to the 2018 PISA (OECD PISA results, 2018) survey results, the **average performance** in the understanding **of mathematics and science for most European countries is quite low.** Even worse, they even tend not to progress. According to another **OECD's PISA survey, it was observed in 2019 that around 1 in 4 pupils did not have the basic level in science (22%) or mathematics (24%)** (OECD PISA results, 2019), which reflects significant shortcomings.

At the same time, there is a **significant gap between girls and boys.** In fact, overall girls have an equivalent level to that of boys, but fewer of them aspire to continue their studies in these fields and prefer to choose a literary or biology branch. This is explained in particular by stereotypes anchored in our society which make girls believe, wrongly, that they would be less talented or not legitimate to pursue a career in scientific fields in general (Libération newspaper Feb 2021). Another point that explains the self-censorship of girls is that there are few examples of women in these fields.

This state of affairs is worrying, especially when we consider that the subjects of science, technology, engineering and **mathematics (STEM) are set to occupy an increasingly large place in society** and the professions of the future. Indeed, according to the think tank IFTF, the Institute for the Future, due to technological development, many professions are set to disappear in favour (IFTF 2023 "Careers of the future") of new ones that do not exist yet and which will be mainly closely linked to new technologies.

What are **the obstacles** students encounter when learning these subjects, particularly mathematics? Several things come into play and it is essential to examine this phenomenon in more detail.

Pupils often feel disinterested in mathematics and this can be explained in particular because they think that the concepts they study are useless. This reasoning also echoes prejudices that have been rooted for a long time now, such as "If you're bad at math, that's normal" or, "What's the point of doing math?".

Another reason is that **STEM subjects**, particularly mathematics, are **often presented as abstract and difficult to learn.** For learners, mathematics has a usefulness that seems too distant from real life.

In addition, we must not neglect **mathematical anxiety**, which is a major obstacle in the acquisition of knowledge. This refers to an emotional matter and not just a cognitive aspect. **This anxiety can lead to material avoidance and underperformance** (Webzine IDELLO, 2020).

Lack of motivation to learn mathematics can also be partly explained by the fact that **pedagogical approaches may be inappropriate**. In fact, our conventional education system is no longer in harmony with the changing landscape of our environment. **Often excessively theoretical, it seems to lack practical applications that could effectively convey learning,** render these subjects captivating, and ignite students' enthusiasm for acquiring knowledge.

It is also **essential to take into consideration all learner profiles,** including those with learning difficulties which, according to studies, represent approximately 15 to 20% of the European population (INSERM Science for health, 2019). We know from several studies that **the learning method used has an important influence on aptitudes, motivation and learning ability** (Marylou Britt in Dumas, 2018).

Students' needs are ever-changing, and teaching methods must meet those needs. In a constantly changing world, the traditional education model must set an example by providing relevant education that will allow learners to acquire solid skills in STEM subjects that will serve them throughout their personal and professional lives.

It seems essential to **provide concrete solutions that can improve the motivation** of learners and their skills.

Here are some **possible ways** to promote the learning of mathematics and more generally of STEM:

- **Apprehend the learning of this subject in a global way** by associating it with other disciplines such as history, heritage and art which is a good tool for reflection and creativity but also the visual support which allows illustration of certain concepts.
- **Set up practical experiments** using different materials, which will promote pupil engagement because they consider these creative activities to be fun and therefore attractive.
- **Integrate new digital technologies**. In addition to being in direct contact with their generation, this will give an additional enthusiasm to apprehend new knowledge.
- **Set up activities in small groups** such as walks or in situ to help the exchange of ideas, reflection and critical thinking.
- **Get out of the classic role of the teacher and be a guide** instead to the pupils. Show kindness, listen, get to know them and value them.
- **Use the environment** to give meaning to learning and make use of **different intelligences.**

As the Mathematician Didier Dacunha-Castelle mentioned in 2015, (Cairn info, 2016)

"Mathematics is about a lot of fields but at college they do not seem to be in touch with the outside world because too closed in on themselves"

Based on these observations, we wish in **this guide to shed global light on the contribution of heritage and its architecture in the learning of mathematical notions** by relying on the contribution of innovative pedagogical approaches.

This is indeed the main objective of our VisitMath project to promote and offer mathematical visits (digital or not) of major European cities and to allow teachers to create their own mathematical visits to cities, villages, campuses...

First, we will explore the links existing since antiquity between architecture and mathematics, and we will also show to what extent, heritage, thanks to a practical approach can make the abstract concrete and thus arouse enthusiasm among learners.

Secondly, we will focus on the benefits of using alternative approaches in education, such as the contribution of games and digital tools, which can improve the motivation of learners as well as a better acquisition of mathematical knowledge in particular.

Finally, in a desire for learning for all, we will show the significant interest of these approaches for inclusion in order to allow all learner profiles and in particular those suffering from learning difficulties, to access mathematical knowledge in a flexible and adapted way.

Finally, we will focus on examples of mathematical visits that already exist in European cities in order to show our sources of inspirations and provide examples of similar initiatives.

2. How are math and architecture related and used in education?

2.1 The links between architecture and mathematics

The VisitMath project aims at creating innovative and practical approaches to teaching mathematics by exploring our surroundings, landscapes, and cities through European architecture. It will allow pupils to understand and appreciate mathematics differently by giving it a more concrete side and increase their engagement by highlighting the diversity and richness of European architecture.

Architecture and mathematics are closely linked, and this relationship has existed for centuries. Mathematics has played a crucial role in the creation of some of the world's most iconic structures, from ancient temples to modern skyscrapers. An obvious question then arises: **How are mathematics and architecture related?** That's what we are going to answer in this section. It will provide an overview of the **historical background** of this relationship and discuss the use of mathematical principles such as **geometry, symmetry, and proportion** in architectural design.

2.1.1 Historical Background

How ancient civilisations used mathematics in architecture

"One of the lasting contributions ancient cultures have made to modern life is architecture, both in terms of surviving monuments and their influence on contemporary buildings around the world" (Cartwrigh, 2019).

The **use of mathematics in architecture** can be traced **back to ancient civilisations.** It was used as a tool to create structures that were both aesthetically pleasing and structurally sound, such as pyramids, temples, and monuments. From Egyptian culture to Mayan, mathematics played an important role in the construction of these ancient architectural wonders.

The ancient Egyptians made significant contributions to mathematics. The construction of the pyramids, in particular, required advanced mathematical knowledge and precision. For example, the Egyptians used a system of measurement called the "cubit," which was based on the length of a person's arm, to ensure consistency in their construction. The cubit (which is divided into palms, fingers, etc.) is probably one of the earliest known units to measure length.

The Maya (pre-Columbian civilisation of Central America) had the most sophisticated system of mathematics of any ancient civilisation in the Americas. Indeed, they were among the first to implement the concept of zero, allowing them to calculate large numbers. The Maya employed the vigesimal system, meaning their number system was based on the number 20, using both their fingers and toes for counting. Their number system allowed them to make extremely precise astronomical predictions, tracking the movements of the sun, moon, stars and planets. The Mayas' knowledge of mathematics and astronomy allowed them to build monuments, such as their well-known temples.

The use of mathematics in architecture is a significant contribution of ancient cultures

1 Ph [illegible] lash

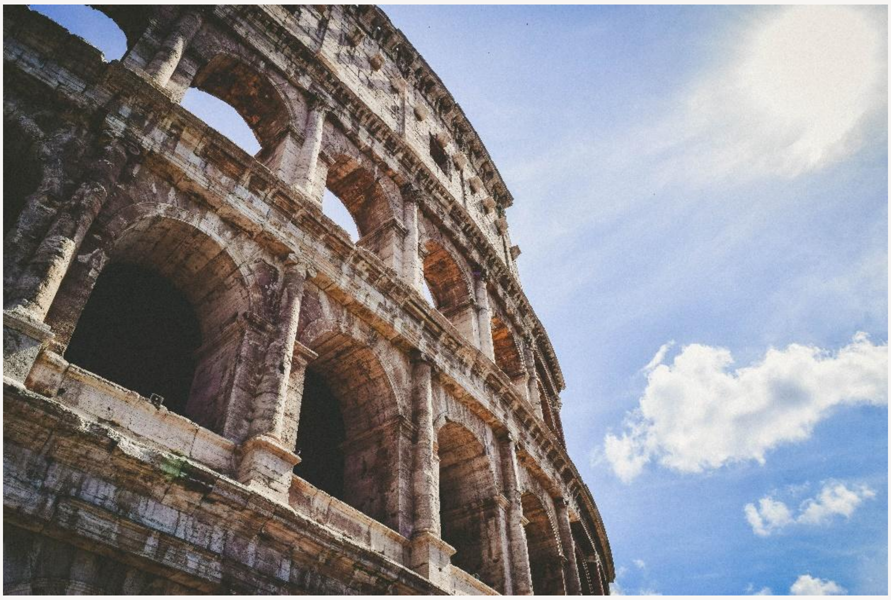
to modern life. Ancient civilisations' advanced mathematical knowledge and precision allowed the creation of impressive structures that still stand today, such as Egyptian pyramids and Maya temples. The contributions of ancient cultures in mathematics and architecture continue to inspire and inform modern practices, showing that the legacy of these civilisations endures even thousands of years later.

Case study: Greeks and Romans

The Greeks and Romans were two of the most influential ancient civilisations when it came to the use of mathematics in architecture.

The Romans

Ancient Roman architects were able to create structures that were aesthetically pleasing, structurally sound, and durable by using mathematical principles and calculations.



2 Photo by Jace & Afsoon on Unsplash

One of the key mathematical concepts used in ancient Roman architecture was geometry. The Romans had a deep understanding of geometry, and they used it extensively in their building designs. For example, the use of the arch, one of the defining features of Roman architecture, was made possible by the development of geometry. The arch is a curved structure capable of supporting large amounts of weight. It was used extensively in Roman buildings, including aqueducts, bridges, and amphitheatres.

In addition, the Romans also used mathematical calculations to ensure the structural integrity of their buildings. For example, the arch was designed using mathematical calculations to ensure it could support the weight of the building above it.

The use of mathematics played a crucial role in the design and construction of ancient Roman architecture. From the use of geometry to the careful consideration of proportion and the use of mathematical calculations to ensure structural integrity, the Romans relied heavily on mathematical principles to create buildings that were aesthetically pleasing, structurally sound, and durable.

The Greeks

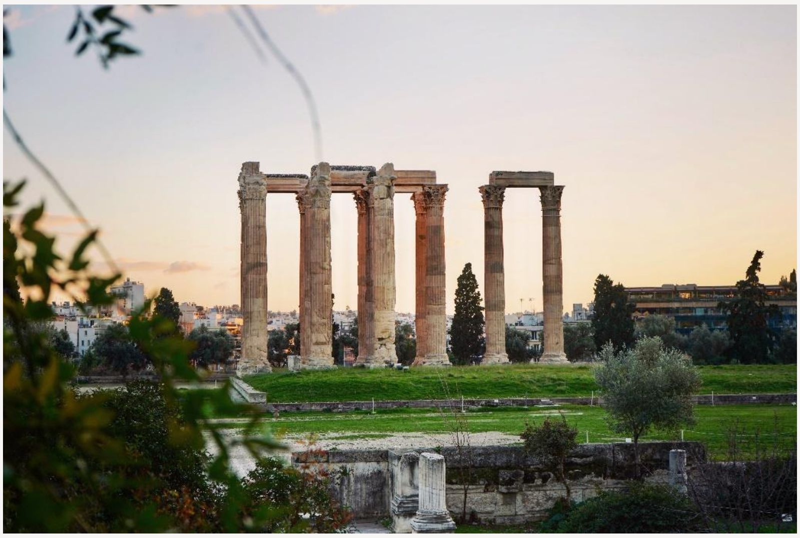
The Greeks used mathematics in architecture to design and construct some of their most iconic structures. They used mathematical principles to create beautiful and structurally sound buildings.

Greek architects used mathematics to ensure that their buildings were symmetrical and proportionate. The Greeks believed that symmetry and proportion were key elements of beauty and perfection and incorporated these principles into their designs. They used a variety of mathematical concepts to achieve symmetry and proportion in their architecture, including the use of the golden ratio, a mathematical ratio found in nature that is believed to be aesthetically pleasing.

3 Photo by Spencer Davis on Unsplash

One of the most famous examples of the use of mathematics in Greek architecture is the Parthenon, which was built in the 5th century BCE. The architects who designed the Parthenon used mathematical principles to ensure that the building was perfectly symmetrical and proportionate. For example, the distance between the columns of the Parthenon was carefully calculated to create an illusion of perfection, making the building appear perfectly straight from a distance.

Another example of the use of mathematics in Greek architecture is the Temple of Olympian Zeus, which was also built in the 5th century BCE. The temple was designed to be one of the largest buildings in the ancient world, and the architects used mathematical principles to ensure that the building was structurally sound. For example, the temple's columns were carefully spaced and angled to ensure that the weight of the building was evenly distributed.



4 Photo by Yang Yang on Unsplash

The Greeks and Romans were two of the most influential ancient civilisations when it came to the use of mathematics in architecture. Both civilisations used mathematical principles to create beautiful and structurally sound buildings, leaving a lasting legacy in modern architecture.

2.1.2 Geometry in Architecture

Geometry and patterns have played a significant role in architecture for centuries. They have been used to create visually stunning and structurally sound buildings. Geometric shapes and patterns can also be used to create interesting and complex designs.

One of the most common geometric shapes used in architecture is the rectangle. The use of rectangles allows for a building to be constructed with straight lines and right angles, which can help to create a sense of stability and symmetry, as in skyscrapers for example.

5 Photo by Sean Pollock on Unsplash

Another common geometric shape used in architecture is the circle. For example, circles can be seen in the design of domes, arches, and columns. The use of circles can help create a sense of unity and harmony and provide structural support.

In addition to creating visually stunning buildings, using geometry in architecture can also help to improve the functionality and efficiency of a building.

Overall, the use of geometry in architecture has been a constant throughout history. Using these design principles has helped to create some of the world's most beautiful and iconic buildings.

Case study: Casa Batlló

6 Gaudi, Photo by Sara Darcaj on Unsplash

A good example of using geometry and pattern in European architecture is the Casa Batlló in Barcelona, Spain.

Its façade features a series of undulating lines and organic shapes inspired by the curves and contours of natural forms. These irregular shapes and patterns create a sense of fluidity and movement.

One of the most striking features of the building is its use of colourful ceramic tiles, which are arranged in a mosaic pattern on the façade. The tiles are arranged in a series of overlapping scales, which create a sense of depth and texture, and reflect the play of light and shadow on the building's surface.

The Casa Batlló is just one of many examples of innovative use of geometry and pattern in architecture.

2.1.3 Symmetry in Architecture

Symmetry is a design principle that has been used in architecture for centuries. It is the idea that two halves of a building should be equal in size, shape, and proportion. Symmetrical buildings are often seen as pleasing to the eye, and they create a sense of balance and harmony.

Symmetry is not only used in grand and famous buildings. It is also used in everyday buildings, such as houses and office buildings. The use of symmetry in these constructions can create a sense of calm and order, which is important in creating a comfortable living or working environment.

Symmetry is a timeless design principle that has been used in architecture for centuries. Whether used in grand buildings or everyday structures, symmetry is important in creating a beautiful and functional space.

Case study: Palace of Versailles

A famous example of symmetry in architecture is the Palace of Versailles in France and its park. This palace was built in the 17^{th} century and is known for its grandeur and opulence.

7 Photo by Armand Khoury on Unsplash

The palace is symmetrical in design, with a central axis that runs through the entire building. The use of symmetry in the Palace of Versailles gives it a sense of order and balance, which was important in the Baroque era. The building's main facade is divided into three symmetrical parts, with identical windows, columns, and other decorative elements.

8 Photo by Mathias Reding on Unsplash

2.1.4 Proportions in Architecture

Proportions in architecture refer to the relationship between different elements of a building, such as its height, width, and depth. It is a fundamental aspect of architectural design, and the use of well-balanced proportions can greatly enhance a building's aesthetic appeal and functionality.

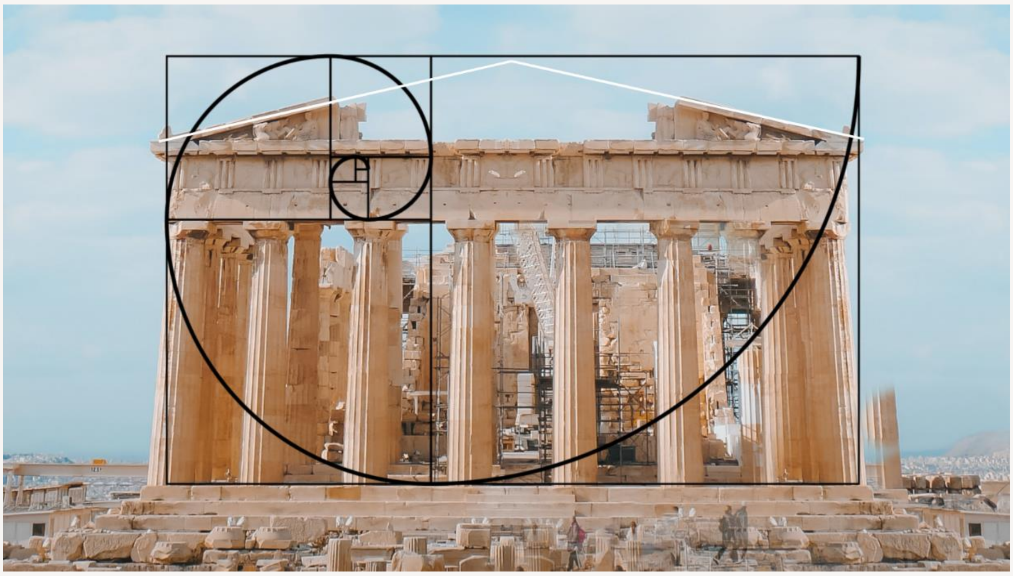
For example, a building with well-proportioned windows can provide an optimal balance of natural light and ventilation, while a building with poorly-proportioned windows can lead to problems with heating and cooling.

In addition to their functional benefits, proportions have symbolic and emotional significance in architecture. For example, the use of certain proportions can evoke a sense of stability, strength, and order, while others can create a sense of movement, fluidity, and dynamism.

Case study: The Golden Ratio

The mathematical principles behind proportional design are often based on the Golden Ratio. In brief, the Golden Ratio (approximately 1.618), also known as phi (φ), is a mathematical concept that describes the relationship between two quantities. It can be found in many patterns and shapes in nature, art, and architecture.

In architecture, the Golden Ratio seems to be used since ancient times in the creation of buildings with a harmonious and balanced appearance. The ratio can be found in various elements of architecture, such as the proportions of windows, doors, and columns, as well as the layout of entire buildings.



9 Representation of the golden number, Photo by Pat Whelen on Unsplash

Mathematical notions - such as geometry, symmetry and proportions among others - undoubtedly form the basis of any architectural construction. The relationship between mathematics and architecture has been linked for centuries, as evidenced by ancient civilisations such as the Egyptians and Mayans. The Greeks and Romans also used mathematical principles to create some of the most iconic structures of their time. We can only appreciate the contributions of ancient civilisations and continue to be inspired by their legacy in modern architecture and mathematics.

The VisitMath project aims to explore this relationship further by using European architecture to teach mathematics in a more practical and engaging way, allowing pupils to appreciate the diversity and richness of European architecture while understanding the mathematical principles that underpin it.

2.2 The use of architecture on math learning

2.2.1 Motivation linked to the practical approach of mathematics.

Are mathematics not motivating anymore? In 2018, the PISA survey showed that several European countries such as France, Italy, Spain and Finland faced difficulties in teaching STEAM. Students are either not performing as well as they used to according to the survey's standards, or students perform below the average 500-point score. The 2019 TIMSS survey observed the same phenomenon in Belgium and Denmark too, among others. In addition, the PISA survey estimated that about 24% of youngsters globally did not master the minimum mathematics knowledge.

a. Theoretical background

Evidence implies that a practical approach of mathematics has two main benefits: it increases pupils' motivation and their results in the subject. This theory has been developed for quite some time: John Dewey (1859-1952) has argued for his whole career for a more **experience-based pedagogy**, one that would focus on theory as much as on practice. Focusing on a more practical approach of mathematics would show pupils that what they learn has a real-world application – and that mathematics can be fun, too! Since few students are interested in mathematics today, researchers tend to emphasise the importance of promoting the beneficial aspects of the subject in our everyday life. According to Abramovich (2019), motivation comes from action and the familiarity of the learner with the studied topic. Giving a pupil an exercise based on something they are familiar with – local buildings for example – can remove the difficulty of abstraction.

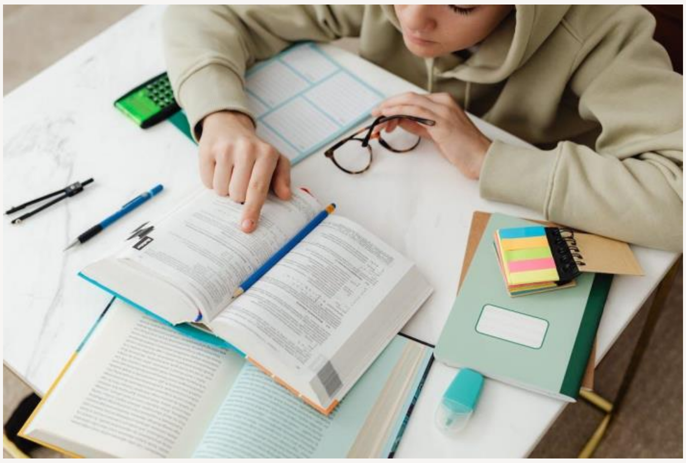
b. Ways to motivate pupils

The practical approach of mathematics should be compelling for students on both levels of motivation: extrinsic and intrinsic motivation. The gamified aspect of the project will increase the eagerness of students to succeed and win the game, while making them interested enough in the subject that they will want to learn more out of sheer curiosity.

Motivation is indeed achieved via the pupils' sense of utility. Since mathematics is often described as too abstract by teenagers, showing them that there are many applications in the real world can potentially help them to change their minds (DfE, 2020). Anchoring mathematics **back into everyday life** has a positive impact on the perception of the subject by the pupils since some learners are lost when they are only given abstract examples. However, this practical approach must not become a distraction for the pupils, and it is crucial that they understand what they have learned beyond the scope of the game . Indeed, one of the main risks of the practical approach is to keep the learners' interests up only for the duration of the course (Abrahams and Sharpe, 2010). This can be achieved by reminding all pupils of why the subjects they study are important, and where they can be found – for example, in architecture.

Motivation and results are heavily linked. Teenagers need **clear goals** and the tools to achieve them, as well as a **clear learning and teaching methods.** Our method will make sure that all teenagers have enough information to experience, at the same time, the practical approach of the use of mathematics in the city, and a clear set of goals that will guide them towards a reward – winning the game. The final achievement (completing the treasure hunt) will be obtainable after a series of smaller-scale exercises which aim at maintaining the teenagers' motivation (OECD, 2017). Our objective is for students to realise that they have successfully completed several mathematics exercises that took place in the real world, which will hopefully keep their motivation up for the rest of the school year(s).

c. Gender equality and professional life



10 Pupil working on math exercises, Grabowska (n.d.)

The practical and gamified approach should also help to remove some barriers of traditional school teaching. First, the alternative method can help include girls more by showing them what they can achieve when studying STEAM, as well as by giving them examples of women working in this field. This approach could also help reengage boys who often develop an attitude of defiance toward the teacher's authority. (OECD, 2017).

Finally, this approach can also help pupils choose a **STEAM-related career path** in the future. Since many children do not consider mathematics or physics to be interesting, European countries currently have trouble recruiting engineers due to the lack of candidates. In France, a study argued that 25% of engineering positions were left empty due to the lack of candidates (Studyrama, 2022). This issue is also visible in Italy (EURES, n.d.) and in Belgium (Manpower, 2019). By giving teenagers more opportunities to see what maths can create and promoting interest in STEAM we hope to motivate them to apply for those jobs.

2.2.2 Why architecture can be engaging

In the scope of the practical approach of mathematics, our focus will be on what can be found in the city, which means architecture in most cases. Architecture offers several benefits to our approach: it is visible and palpable, it is based on mathematics and can inspire applications in many different mathematical fields such as arithmetic and geometry, and it can provide enough material to link mathematics to other school subjects. As we stated before, the practical approach is engaging in itself, but architecture can be used to drastically increase the pupils' interest in the matter.

a. Architecture and mathematics at school

Indeed, there is a clear difference between studying practical mathematics via stereotyped exercises about unreal quantities of watermelons at a supermarket checkout and looking at building around us. One can easily imagine exercises that would focus on the tower of Pisa or the Parthenon. Actually, many more buildings, streets or installations can be used for mathematics, even in less touristic areas. In addition, if the pupils like the idea, the teachers can still rely on architecture to create more mathematics exercises: the main part of this challenge is to ensure that the pupils see this as applied mathematics, and not as an imaginary situation.

In addition, one of the greatest advantages of using architecture in mathematics classes is the variety of topics that can be studied. **Geometry** is of course an obvious subject to study: the angles and the symmetry between different parts of a monument (think about the gardens of the Palace of Versailles for example) come to mind, but **arithmetic** can also be used. Younger learners can discover **measures** (height, length, volume) with buildings or roads and study time, while more advanced learners can learn more about the golden number and the **golden ratio** using the Parthenon as a reference, as some say.

11 City by night, Pasaric (n.d.)

b. Cross-curricular teaching

As stated earlier, the possibility of cross-curricular content can make mathematics more interesting and architecture lends itself very well to this kind of method. While History comes to mind at first (when talking about specific monuments that were built at a certain time), other topics can be addressed. For example, a pair-up with Economics can be used to study fractions and percentages, geography can be used to study Statistics too. And let's not forget about Art and the possibilities of mathematical drawings or a more advanced study of soundwaves in a theatrical setting, where one could link mathematics, physics and music. Since Architecture can be used as a cornerstone for many activities around mathematics, teachers will have the opportunity to find a variety of ways to reach all of their pupils.

c. Architecture is everywhere

The use of architecture in mathematics classes can also take the school environment into account. After all, when one thinks about architecture, the first images that come to mind are famous buildings in big cities. Yet, architecture can be found everywhere, even in villages and at the countryside. This means that rural schools can also participate in the testing of the project by studying the layout of their surroundings and discovering their local architecture. And if the material is not interesting enough or too redundant, the pupils themselves can become actors of imagining what they could create, which gives them the role of budding architects! (Philippot, 2019). As we stated before, this concrete approach will have a positive impact on the teenagers' motivation since they will allow them to understand the mathematical concepts more easily.

3. The benefits of using alternative approaches in education, like gamification and e-books, for all types of learners

3.1 Gamification and e-books for learning math

3.1.1 The importance of games in our everyday life

As a subject, mathematics is not usually represented as being particularly engaging. It is often seen as purely theoretical with little to no real-life application – an issue we tackled in the previous part – but there needs to be more than just an improvement of the content of the subject. The way of teaching needs to be adapted too, so that the learners can understand the subject better.

a. Pedagogic background

Indeed, pedagogy theory has progressively evolved from theory-based education to a more practice-based one, with the learner becoming more and more active in class (Abramovich, 2019). The goal of this method is to have the learner try things and fail, with the teacher sometimes acting more like a "safety net", a resource that is available for the pupil to consult if things become too difficult. However, this method of teaching only works if the group is already motivated to achieve the task, and this is where mathematics classes sometimes struggle. When the teacher acts as a facilitator (the most "free" teaching style), pupils are left with their tasks and expected to progress with little help. If the subject is not compelling, this method may not lead to a very productive outcome, which is why this project intends to gamify the architecture-themed math lessons.

12 Cities: Skylines board game, Thames and Cosmos, 2019

b. How to create a good game-based course

So why games? First, games make pupils actors of their learning. The teacher is not in a position of control, so the learners are freer to act and try new things. Contrary to more traditional exercises, games intend to be more compelling – they must be fun. Pupils must forget that they are working as they progress through the steps of the game: the game mechanics must feel like more than an excuse to practice a specific topic. The gamified approach must not be a "skin" that could fit any kind of exercise, but rather a set of mechanics that fits specific situations. Still, there is no perfect recipe to create a perfect game: the design of the game can be **top-down** (think about the setting that you want to give first) or **bottom-up** (think about the topics that you want to practice first). Still, a game needs to be fun in the first place, and just having an interesting topic will not be enough to compel players (Mark Rosewater in GDC, 2016). Content must be organised to create an environment that players will want to experience again.

Having learners adhere to a gaming system does not require exactly the same parameters as having people adhere to a brand new game since pupils have to go to school: the goal is to engage them to create a fun experience but not to convince them that they should do it for a certain amount of time. Still, in order to ensure that they enjoy the gamified lesson, some form of **competition** must exist, either between the players themselves (everyone is by themselves or in small groups, against the others) or against the game (collaboration to solve a problem) (Karin Tsai in GDC, 2021).

13 Game controller, juicy_fish (n.d.)

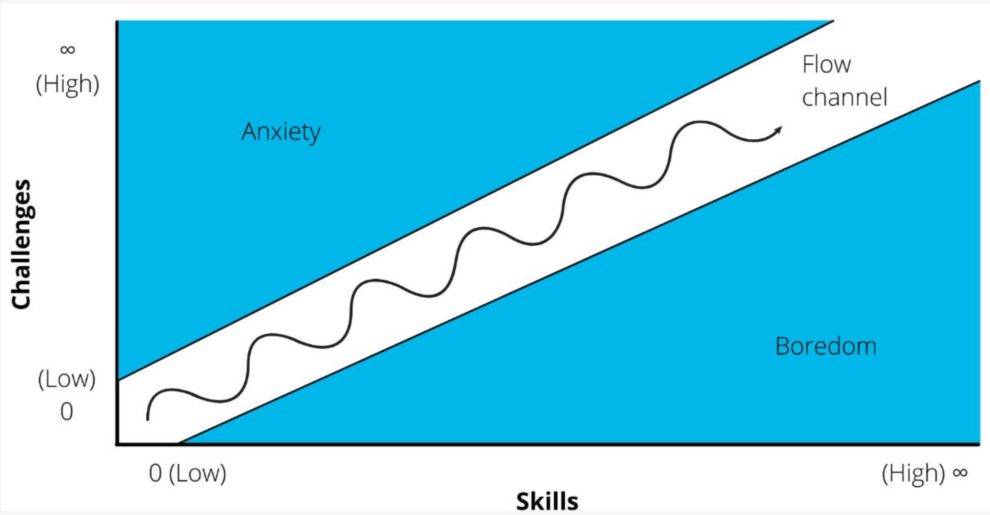
However, since there are as many player profiles as there are pupils (and that is a lot!), some aspects of the game need to be taken into account before it begins in order to keep everyone motivated. The goal is to create a cheerful atmosphere, and not to make anyone feel less valuable. Since the goal of a game is to win, those who do not succeed must not be left out. Comparing the players between themselves, for example, can lead to resentment (Nicholson, 2013): if all success and failures are displayed on a public screen, the most probable outcome will be the lack of motivation of those who trail behind the first three or four players (Lundin & Melkersson, 2022). However, the competitive aspect of this method also tends to motivate many pupils, while the badge approach does not always do the trick. As game masters, it is our job to find the middle ground to motivate all learners to participate and do their best.

c. Game structure

So, how does one create an efficient game structure? First, pupils must know how the game works. A tutorial, or an easy task, can set the players in motion: the rules must be clear from the starting point. Then, the difficulty should increase gradually: in the case of a mathematical exercise, the first question should be rather easy to answer in order not to discourage the player and become more and more challenging. If the questions are linked, this is obviously a big bonus for consistency. As the pupils answer, the game should guide them towards the end.

Reaching the end of the game must **mean something.** This is why rewards are important: they foster motivation. The reward must look interesting, so it needs to give learners a feeling of accomplishment when it is reached, which will keep them motivated in the future. Still, collecting frequent rewards with regular progression updates can engage pupils more. The update must be meaningful and encouraging ("Congratulations, you have reached...") and the pupils must know when they have reached the final goal.

When it comes to difficulty, the game must be **challenging** – not too easy, not too difficult. This will create the right amount of engagement, since easy games are not taken seriously and difficult games can be discouraging: being stuck (especially at the beginning of the game) can lead to disinvolvement, cheating or abandonment. Although there are many kinds of players (some players like difficult games where they fail over and over again before finally succeeding, but some others like to play easy games that allow them to do something else at the same time), this game must correspond to everyone at the same time and thus be "middle-ground". Mihaly Csikszentmihalyi defined the perfect level of difficulty of any given task in his book *Beyond boredom and anxiety* in 1975. The **Flow**, as he calls it, is what motivates someone to accomplish a given task. This flow is the level of enjoyment, which can only happen if the game is challenging enough but not too much and, although this state depends on the player, the teacher must be careful of how to plan their game so that everyone involved has a good time playing it.



14 Mihaly Csikszentmihalyi's flow theory from the Gaming for skills project

d. Mathematics in citizenship training

Games allow learners to train additional skills especially when they play in groups. When mathematics is often based on individual exercises and little to no communication, games enable pupils to work in groups and share their ideas. This way, they practice their **soft skills** alongside their mathematic skills (hard skills). Soft skills are "abilities that relate to how you work and interact with other people" (Herrity, 2023). They are usually not taught at school and are, actually, rarely trained. They consist of communication skills, teamwork, task management or problem-solving, among many others. They define us as employees and citizens, and are great assets at school and at work. Pupils do not usually realise that they are practising their skills when they work in groups, but making them actors of their own learning always has a positive impact on them. This kind of activity can help them know more about themselves and their classmates!

3.1.2 Guidebooks for games

a. What is a guidebook?

A guidebook is "a book that gives information for visitors about a place, such as a city or country" (Cambridge Dictionary). Its goal is to show the important parts of a place people want to visit, or to show details about a structural element.

15 Kalimnos climbing guidebook, 2006

In our case, the guidebook will feature the **locations** that pupils are expected to visit and provide them with indications on how to reach them. They will also contain the mathematical **riddles** that the pupils will need to solve to access the next one. Of course, the role of a guidebook is not to reveal everything about the place but rather act as a complementary tool that will enrich the touring experience. It has more of an advertising role than a descriptive one.

b. Why are guidebooks useful?

As mentioned before, guidebooks showcase the main attractions of the city. Their role is to select what is important to guide the tourist to that place. In the case of a treasure hunt happening in an "open world", a place with almost no boundaries, the guidebook must act as a way to keep the learners on the right tracks. It acts as a paper escape room or RPG game master, in a way as the players must be guided to enjoy the experience, otherwise they might get lost. And since some cities can be quite vast, having a guideline will certainly not harm the pupils!

Additionally, guidebooks usually provide information about the places to see. They will help pupils be more **immersed** in their treasure hunt: it must not feel like another typical school work, like they are answering questions or solving problems. The cultural aspect of the guidebook is primordial to ensure everyone is having a good time while practising their math skills. In smaller towns or in the countryside, when there are no major monuments, try to add **fun facts**, or highlight interesting **things to do** in the surroundings. What is the history of the place? Are there nice shops in the area? A café, or a restaurant that is worth mentioning? What do people do here? Can people hike or ride a bike in the surroundings? Is there a landscape to see? All of these ideas can help market your small town and make it more attractive to teenagers!

The guidebook can be used as a pedagogical tool. It must be created with problem-solving in mind, so the pupils must be able to use it when they are answering questions, either by reading it to find information or writing on it. It can also contain hints about how to solve problems or some advice from the teacher.

c. The advantage of using an eBook

eBooks – short for electronic books – are digital texts that can be read on electronic devices. Although the content remains the same, eBooks can allow a more fun and inclusive experience.

16 eBook user, Cameron (n.d.)

The first major upside is the fact that eBooks are interactive. Writing on a piece of paper can become messy, especially when content needs to be erased and re-written. Pupils can write annotations on the document, delete them and add some more without interfering with its readability. They can also **search** directly within the document: a treasure hunt guidebook could contain many steps, and pupils could get lost. The research function will help them find what they are looking more quickly. Then, it also allows teachers to store more information, or to hide some until the right time has come. Interactivity in an eBook can include clicking on items to reveal information, moving items from one place to another or even, if necessary, add some sound!

In addition, since eBooks are digital, teachers and pupils can **upload pictures** or videos on their devices. For some specific exercises (let's say one about symmetry) they can take a picture of the building structure and answer the exercise on their eBook. As we mentioned beforehand, the first role of a guidebook is to show information about important places: here, information can be backed up with external content, thus enhancing the feeling of an educational tour rather than a mere mathematics class.

Inclusivity is made by the user's possibility to **zoom-in** or change the **screen's brightness** (Digital Unit, 2022). This is particularly important for visually-impaired people, but can prove useful even when exploring the outside in the winter for example, when night is falling. Pupils who struggle to read the displayed information can also listen to the guidelines since audio transcription is available on those devices.

Such parameters must be taken into account when designing content accessible to all. Indeed, using eBooks in this project is part of our commitment to include all children in gamified content.

3.2 Inclusion

What does it mean to be inclusive? Inclusion, especially in education, is not only about adapting materials in order for all profiles of pupils to access these materials and be able to follow everything. It also ensures that the pupils encountering difficulties know how to use those materials. Moreover, inclusion is also about changing how things are organised at school to suit the learners' needs without other pupils' learning being affected. It is basically **making learning flexible**, and it requires constant thinking about the teacher's practice and pupils' way of acquiring knowledge.

The VisitMath project aims to make STEM subjects - mathematics in particular – accessible to all pupils. The project will address social inclusion by making sure no one is left behind especially the pupils who typically feel less confident and engaged with sciences or mathematics, such as children with Specific Learning Disorders (SLD) or any pupil facing difficulties in mathematics and science. It will allow for a comprehensive and inclusive education.

People with SLD are thought to represent around 9-12% of the total population (European Dyslexia Association, n.d.) and are navigating through life in a largely non-"dys" friendly world. The proportion of pupils who could potentially have trouble following non-adapted materials if they are not adapted is thus higher than most people would expect. As education should be accessible to all pupils, we will endeavour to adapt our content as much as possible.

3.2.1 What are SLDs?

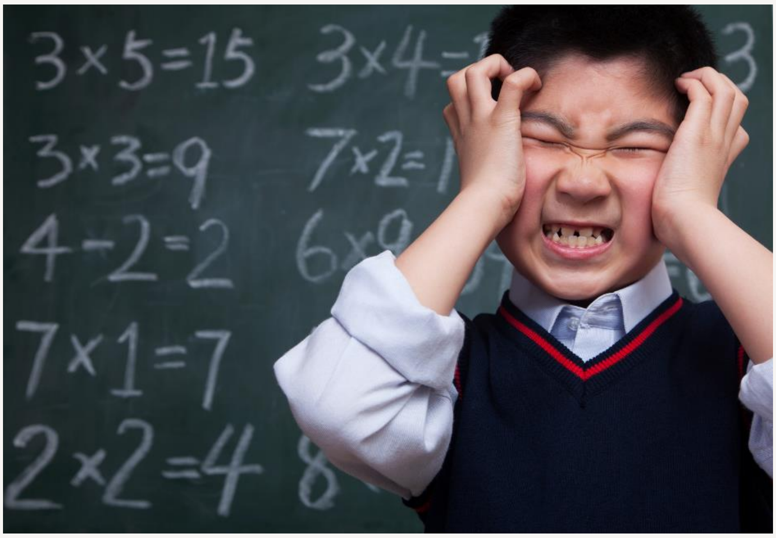
First, it is important to understand that SLDs are not stemming from a physical impairment, an emotional disturbance, or a socioeconomic, environmental or cultural disadvantage. SLDs have a **neurobiological cause** affecting how the brain processes information: essientally, how it receives, integrates, retains, and expresses information. This can **disturb the cognitive development of learning abilities,** such as reading, writing, speaking, doing mathematics, or planning and coordinating motor tasks.

Here are the different specific learning disorders as usually listed:

- **Dyslexia** causes difficulties in reading and language-based processing skills. This dys-disorder is the most common, and it is not rare to overlap with another one. Concretely, it affects the pupil's ability to learn to read and write. Those skills are already difficult to acquire, but with the cognitive difficulty of deciphering a text added, they need to double the effort to learn.
- **Dysgraphia** is a specific learning disorder that affects a person's ability to write legibly and coherently. It affects the fine motor skills and the ability to organise thoughts and ideas into written language.
- **Dyscalculia** affects a person's ability to understand numbers and learn mathematical facts. Pupils with dyscalculia often lose track while counting, mistake numbers during operations, and have trouble memorising and recalling mathematical procedures and rules among others. They can also have difficulty telling and estimating time.
- **Dysphasia** concerns the development of oral language and can affect the receptive and/or expressive aspects. In other words, it translates into difficulties with speaking and understanding spoken words.
- **Dyspraxia** causes issues with coordination, movement, language, and speech. It typically affects fine motor skills and muscle control (including eye control), leading to movement and coordination problems, especially hand-eye movements, language, and speech.

- **Attention Deficit Hyperactivity Disorder (ADHD)** is a neurodevelopmental disorder characterised by excessive, pervasive and intrusive inattention, hyperactivity and impulsivity in multiple contexts.

Very often, various dys-difficulties co-exist within the same learner. For example, the European Dyslexia Association considers that 20 to 40% of **people with dyslexia also have dyscalculia** (European Dyslexia Association, n.d.).



17 Source: Canva

The identification of the SLD is critical in order to be able to provide pupils with appropriate support. An unidentified SLD will make learners face greater difficulties, as the specific difficulties they face might not be taken into account by the teachers, educators or parents. Therefore, there is little guarantee that they will receive the appropriate care or have the possibility to go through school with adapted methods or tools.

The earlier the diagnosis is made, the earlier it is possible to work with the learners to compensate for their difficulties, find relevant coping mechanisms, establish a dialogue with the schools, etc.

3.2.2 What's the impact at school?

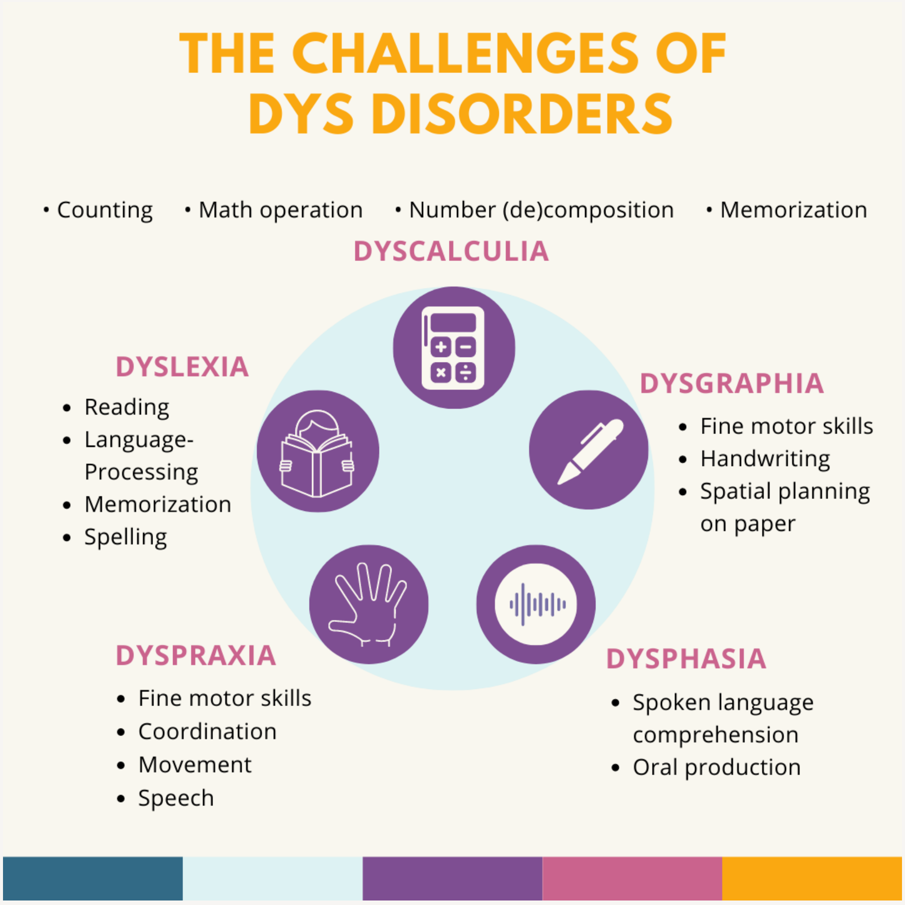
The first real issue that any of these SLDs cause for pupils is that they have to make an extra effort at all times in order to follow the rhythm because their brain is permanently in double task mode. Often, a specific learning disorder can be observed when there is an inability to automatise a type of task that most children and people have automatised rapidly or early on.

The classroom environments can often be overwhelming for dys-pupils. They are usually crowded, busy places that have a lot of distractions.

Each SLD can generate its own set of challenges:

- With Dyslexia, the brain takes longer to connect letters and words with other kinds of knowledge, and this can affect reading fluency, decoding, reading comprehension, recall, writing, spelling, and sometimes speech. Furthermore, at school, dyslexic learners may have difficulties taking notes, lack engagement in reading and writing, misunderstand instructions, etc. As most of our current educational system relies on reading and writing skills, a cognitive disorder affecting those areas can be a real challenge.
- Dysgraphia leads to difficulties remembering specific orthographic combinations, spelling, spatial planning on paper, sequencing sentences into words, composing, or thinking and writing simultaneously, as well as a tendency for overlapping letters, words and inconsistent spacing. Dysgraphia is most noticeable as illegible handwriting.
- Dyscalculia has many consequences on everyday life. Concretely, dyscalculic pupils have trouble manipulating numbers in general, that is to say money, time, quantity, distance, etc. At school, if the material is not adapted, it may translate by performing poorly in tests, becoming easily overwhelmed, and developing math anxiety. Learning STEM with dyscalculia is even more of a challenge than usual.
- Dysphasic pupils typically show difficulties speaking and understanding spoken words, which can be a challenge in the oral exercises and presentations often required in the classroom. This can take the form of difficulty in sequencing sentences into words when heard. It can feel like listening to a foreign language and not knowing when a word ends and when the next begins. Dysphasic people will usually present difficulties with constructing the structure of a sentence or a story.

- **Dyspraxia** affects the ability to perform tasks that require fine or gross motor skills, such as tying shoelaces, writing, or playing sports. People with dyspraxia may also have difficulties with spatial awareness, perception, and organisation, making it difficult to carry out everyday activities.
- In practical terms, **Attention Deficit Disorder with or without Hyperactivity (ADHD)** involves difficulty in remaining seated, concentrating and ignoring external stimuli (noise, light, etc.). Students with ADHD are also more likely to chat in class, or forget their material at home.



3.2.3 The benefits of inclusion

Not only are pupils with SLDs intelligent, full of potential and deserving of an adapted education, but they also have a unique outlook on concepts and life due to how they experience their environment. Individual differences between pupils are a source of richness and diversity. Therefore, it is necessary to be open to dialogue so we can explore what they can offer in terms of ideas, perceptions, and out-of-the-box thinking. Multiple-ways of approaching ideas and problems, as well as out-of-the-box thinking, after all, is what STEM learning encourages.

Opting for alternative methods to include dys-pupils or any learner facing challenges allows them to develop the necessary skills in order to flourish in their personal and yet-to-come professional life. They will be able to function in a society that is not intrinsically adapted to their needs and acquire some tools to help them navigate life more confidently. They will also feel more integrated within the group and will be able to prosper on a personal as well as an academic or professional level.

3.2.4 How to adapt educational material in general

There are several techniques and small reasonable adaptations that can be made and can benefit everyone in the classroom without impeding the general learning process:

- Structure: It is always better to start the lesson with an explicit explanation of the activity, set clear guidelines and subdivide the tasks into clear, small steps if necessary. The use of visual elements to illustrate the concepts and bullet points to structure processes is advised. Make sure to give enough time for each task and that all pupils understand what they have to do beforehand.
- Environment: It needs to be quiet, but with enough multisensory stimuli to allow for in-depth learning. The stimuli should be pertinent to the lesson, without any unnecessary distractions. Space should be uncluttered and not overcrowded to help with the pupils' spatial orientation and their focus. It is also advised to avoid the necessity of long eye movement and to give special support to learners with tasks involving space management.
- Tasks: It is better to multiply the types of exercises to train pupils to process different situations by focusing on one task at a time. For all exercises, it is always better to focus on logic rather than memory. Especially for all tasks involving fine motor skills, try to reduce the number of writing tasks and avoid

difficult manipulations so that the pupils may concentrate on the content of the lessons more than on executing a supporting task.

- **Written materials**: As reading is usually a source of challenge, it is advised to use an adapted font for written guidelines, such as Arial, Century Gothic or OpenDys. The spacing should be 1,5 between the lines, in a font size between 12 and 14. The text should not be justified but rather aligned to the left. The text should also be broken down into smaller paragraphs and clear sentences.

In conclusion, innovative approaches can play an essential role in including a wider scope and variety of pupils. Indeed, they prioritise a more flexible, accessible and understandable approach to learning; maximising the chances of offering a learning method that draws on each individual's strengths, making learning more fluid, sustainable and rewarding for the learner by improving his/her self-esteem; that's precisely the aim of the VisitMath project.

4. Other complementary resources

In order to provide additional information to this guide, we would like to share the initiatives that are part of the same idea of approach, namely the discovery of heritage, mathematics and culture in Europe through various paper chases.

Different formats are available. Here are some examples:

4.1 Mathematical Rallies

These rallies or trails have existed since 1980 and take the form of a walk through the city or the countryside or in the school, allowing pupils to discover the surrounding context while solving mathematical problems. A way to introduce students to real-life experiences that will complement the concepts covered in their school program. Usually, the concepts that are addressed there call for knowledge of geometry and calculation, for example, the height of a building or the volume of water in containeer. They can be adapted to the level of the students so we can include more complex problem-solving such as fractions or statistics.

4.1.1 MathCityMap (ERASMUS+ Project)

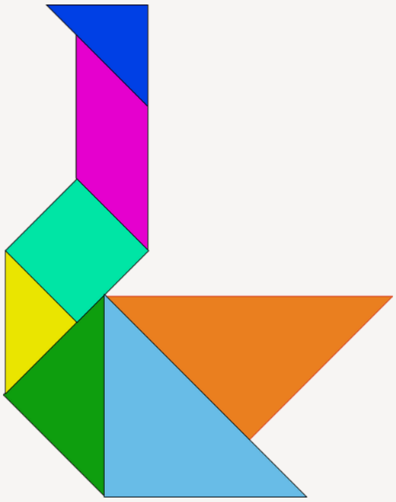
This project created in 2012 continues to evolve over the years. The idea is to submit puzzles that pupils will have to solve by moving to different places. A mobile application has been developed, which allows pupils to be geolocated in real-time. Upstream, the teacher will have defined the methods of answering the questions asked (for example an interval, a fixed number of answers or multiple answers), as well as the level required to be able to solve the puzzles located on the course.

18 Westhafen Tower, credit photo Wikipedia / Pavement On The Old Road In France, credit photo Freepik

If several groups complete the same route at the same time, they can see the score of the other teams, which allows competition between groups. They can also communicate live via the application with their teacher.

4.1.2 IREM of Lille (Research Institute on the Teaching of Mathematics)

In partnership with local actors, this institute has been organising the Mathematics Rally for middle school pupils for several years. It begins in several schools and ends with an academic final. Teams must solve puzzles within a time limit. The puzzles are varied and correspond to all levels of the college course.



19 Puzzle Vector, credit photo Freeimages.com

4.2 Maths walks

4.2.1 M@ths en-vie

In the desire to **develop non-formal pedagogy in the resolution of mathematical problems**, the association M@ths'nCo proposes through this device created in 2016, to anchor mathematics to the reality of pupils. **This collective is made up of teachers and trainers**, each using the tools to engage pupils in an active vision of learning mathematics. These actors then pool their knowledge and their feedback in this collective.

Teachers are encouraged to organise mathematical outings with small groups of pupils. The concepts covered are varied: the numbers visible on road signs, geometric shapes, straight lines thanks to lines drawn on the ground etc... There are so many things to discover outside of the classroom for a direct link with the familiar environment of the pupils.

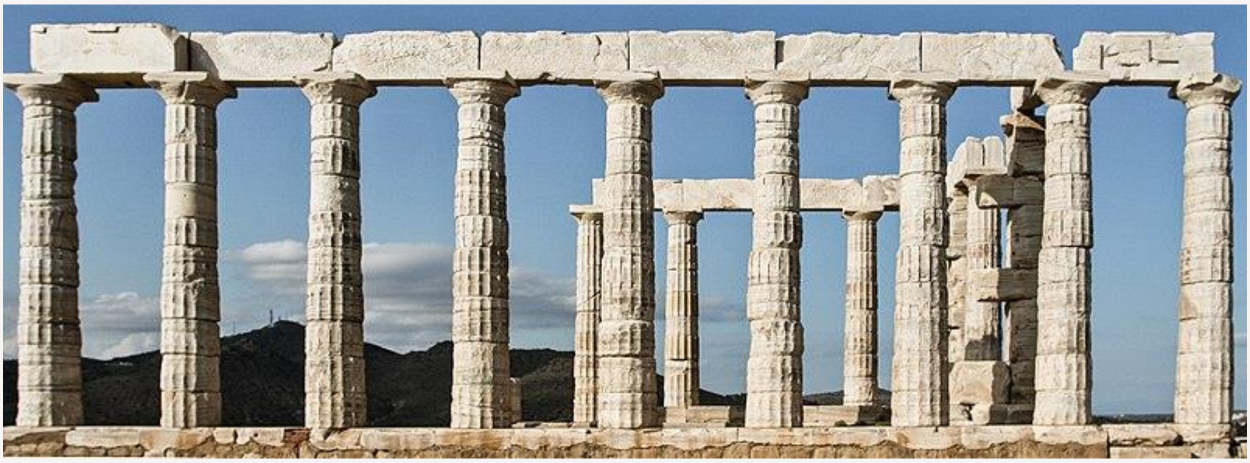
20 Mathematics in the city, credit photo Adobe Stock

4.3 Educationals trips

Some **tourist structures** offer pupils the opportunity to travel across Europe and other countries, tackling a wide variety of subjects and immersive activities.

This is the case of **WorldSrides** which offers educational trips for pupils. It is a way for them to learn about culture, history and heritage, by experimenting through the discovery of different countries.

Their offer includes a STEM trip to Greece for high school pupils**. Discover the culture** of this country and **become familiar with mathematical principles and its innovations**, but also participate in cultural activities and even create their own Olympic games.



21 Temple of Poseidon, credit photo Wikipedia

4.3.1 Objective Science International

Since 1992, this association has been offering **family holidays**, training courses or scientific school outings for everyone and for all levels. **Accompanied by scientific educators**, participants can immerse themselves in unique adventures.

22 Man dressed as a scientist, credit photo Freepik

The proposed **activities are directly linked to scientific research projects**, and the affiliated partners provide state-of-the-art technology equipment. This a fun way to get involved in scientific research, since all the data collected is then shared with professional researchers.

4.3.2 Educational stays

The "sejours-educatifs.org" platform of the education league in France offers a **wide range of stays in France and abroad from kindergarten to high school.** Very diverse themes are proposed there, such as science and experimentation, or developments and scientific progress in architecture, sustainable development, history, citizenship, etc.

4.3.3 Monument Tracker

This application available **on your smartphone**, allows you to locate the buildings in the user's surroundings.

It focuses on the interest of learning about monuments worthy of interest and their history thanks to geolocation but also on the game.

The user can choose these centres of interest in advance or be guided by the application which sends a notification when approaching a memorable place.

Then Monument Tracker offers several choices:

- Audioguides narrated by actors.
- Comments written by journalists.
- Games, in the form of quizzes, challenges or even competitions.
- Thematic visits.

More than **75 destinations are listed around the world** and offer an offline version, so it is not necessary to have an internet network, you just need to download the city map beforehand.

This application can support learning certain mathematical concepts that can be added during the visit to the city.

4.4 Treasure hunts

4.4.1 MyCityHunt

Autonomously thanks to a simple **smartphone**, everyone can access one of the activities offered directly on the internet.

This structure offers a wide choice of activities in many cities in France and Europe by **offering immersion in cities through the implementation of a scenario which promotes immersion thanks to the imagination of role-playing...**

The centre of the city then becomes a real playground for the participants. This is particularly the case in the city of Toulouse in the south of France, where this treasure hunt is presented thanks to the story of a very old manuscript with enigmatic drawings and letters that will lead to a hidden treasure somewhere in the city. Once a role has been assigned to each participant and provided with the smartphone application, the hunt can begin!

4.4.2 Treasure hunt

This structure **brings together known existing treasure hunts in several countries**. They are classified into four categories depending on whether they are permanent, temporary or linked to a particular event.

The themes are very varied and some **even offer real treasures to be won** in the form of money or more or less important prizes depending on the level of difficulty of the puzzles to be found.

23 The treasure chest credit photo ludeek.com

4.5 Paper chases

4.5.1 Bello Monte

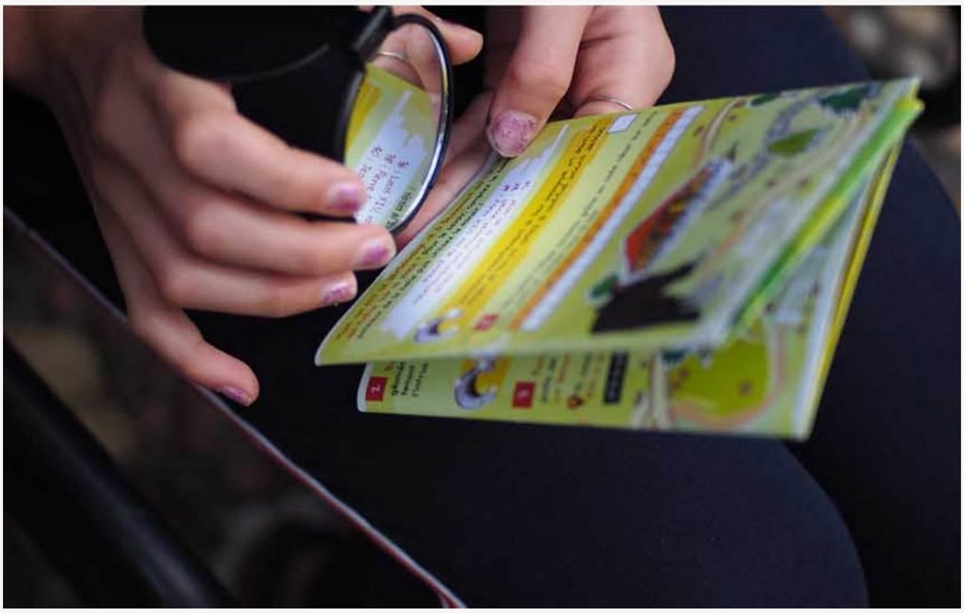
Through the alleys of the **Bastide de Beaumont-de-Lomagne in France**, young and old children can answer the riddles along a route to hope to reach the ultimate riddle and thus accomplish their mission.

Proposed by the "Office intercommunal de la Lomagne tarn-et-garonnaise" and composed of 5 stages, **this mathematics and heritage game gives the possibility of surveying a medieval village, homeland of the mathematician Pierre de Fermat,**

while solving mathematical puzzles.

A pouch comprising complementary accessories such as a 13-knot rope, a compass, a mirror, etc. will help participants meet the challenge.

This game is mainly done in small groups and everyone can be given one of the step missions, to take note of the answers or to be in charge of the accessories of the adventure bag.



24 The magic irons of Bello Monte in Beaumont-de-Lomagne, credit photo Malomagne.com

4.6 Escape games

4.6.1 S'CAPE

This **platform offers a very wide choice of escape games on many subjects** ranging from history, literature, science, sport, mathematics etc...

Also, it is possible and even recommended, to choose the category corresponding to a level, a discipline and a specialty.

For example:

- "college" level,
- discipline or "science" "revision" learning phase.

Its database is extensive and can accommodate a wide range of participants.

Here is an example for an audience of high school pupils in science and technology to discover and deepen notions for a particular topic:

"Alan Turing and the salt marshes"

Script:

Alan Turing discovered a coded message mentioning a bomb hidden in this factory. Locate it and disable it.

The pupils will have to dig in an abandoned factory and then have to find a code lost in the middle of other elements.

Throughout the investigation, there are codes to find, padlocks to open, binary digits to convert...

The puzzles of the game are of variable level which allows to give confidence to the participants and to make the activity attractive and stimulating.

Did you like our tour selection? Find them - and many more - on the internet!

5. Conclusion

Is the reputation of mathematics as an "abstract" or "superfluous" subject justified? In this guide, we have produced many pieces of evidence that suggest quite the opposite. Mathematics is, indeed, all around us, in the buildings we walk past or in the streets we walk on. Teachers can help pupils notice the concepts in their environment. All we, the learners, have to do, is to **pay attention.**

Highlighting the omnipresence of mathematics in our everyday life has several advantages: first, by removing the stereotype of the subject as being too abstract, we can pin back mathematics into our reality. This subject has existed since the Antiquity, perhaps even before, yet nowadays it is considered as not interesting enough. The subject had even been removed from French high schools between 2019 and 2023, except for pupils who wanted to take it as an option. Overall, the European mathematics level was on the decline, according to the 2019 TIMSS survey.

But mathematics is at the core of many, if not all, STEAM-related jobs. STEAM are also part of the European priorities: **mathematics enable analytical thinking** and help develop soft skills such as problem-solving and general curiosity. At the same time, many European countries struggle to find qualified engineers on the job market as stated in recent EURES reports. This lack of employees, alongside the inequalities in gender representation in STEAM learners and workers, have helped us write this guide with the hope that it will be useful for many people and give pupils a taste of mathematics.

Architecture is everywhere, or rather, everything we build was made thanks to architecture, which makes studying buildings and urban planning much more "real". And, since architecture has existed for many centuries, every pupil may be interested in this approach. Whether they are fond of **Ancient Greece theatres, Renaissance palaces or modern skyscrapers**, all of your learners will be able to mentally see what architecture can help build through. Exercises that are related to this topic and then become real-life applications of the mathematical concepts. Whether the class is about symmetry, geometry or proportions, all subjects will sound much more realistic to your pupils and thus will motivate them to succeed.

Although mathematics is made more concrete with the example of architecture, some students may still find this approach boring. Bringing in a gamified approach intends to convince those who are not just there yet. By making mathematics more fun, we aim to convince a larger part of learners to keep learning about the topic at university and in their professional careers. The gamified approach aims to motivate pupils so they perform better in class without even noticing it. Plus, adding digital content will help include those who struggle in class, for example, **pupils with SLDs.**

The various issues that Specific Learning Disorders create are not often addressed at school. Pupils who are affected need extra help to succeed in their scholarships, and providing that help is part of our mission when designing this guide. Indeed, why would we not make mathematics fun for everyone? The digital approach and the **use of eBooks** in particular can help pupils who are struggling to display information as they wish and to organise their lessons in an adaptive way. Having a readable font, easily accessible content and a way to organise tasks will make their time at school more enjoyable.

Finally, in order to diversify your activities, we provided a guide on various activities that are either already focusing on mathematics or that can be adapted to fit your needs as a teacher. Once again: mathematics can be fun, so why not solve puzzles, enjoy recreational mathematic walks or even… go on a **treasure hunt** revolving around mathematics?

Whether you are looking for new ideas for your next mathematics classes or trying to find a way to adapt your courses for all pupils, we hope that this guide has been useful! Do not hesitate to read our other guides if you want to learn more about how to link architecture, mathematics and any other school subject through treasure hunts and many other games!

References

- Abasi, A. U., Ado, I. B. (2014). *Effect of Practical Approach on Basic 7 Mathematics Students' Interest and Performance in Fraction in Uyo Akwa Ibom State of Nigeria*. https://core.ac.uk/download/pdf/234636341.pdf#:~:text=Practical%20approach%20is%20a%20situation%20in%20which%20teachers,patterns%20in%20mathematics%20leading%20to%20rules%20and%20formulae.
- Abrahams, I., Sharpe, R. (2010). Untangling what teachers mean by the motivational value of practical work. *Association for Science Education*. https://core.ac.uk/display/29175721?utm_source=pdf&utm_medium=banner&utm_campaign=pdf-decoration-v1
- Abramovich, S. Grinshpan, A.Z. Milligan, D.L. (2019). Teaching mathematics through concept motivation and action learning, *Education Research International, 19*. https://doi.org/10.1155/2019/3745406
- Abramovich, S. Grinshpan, A.Z. Milligan, D.L. (2019). Teaching mathematics through concept motivation and action learning, *Education Research International, 19*. https://doi.org/10.1155/2019/3745406
- Best, J. (2019). *The Value of Games and Gamification with Mathematics*. https://www.mathletics.com/blog/educators/value-of-games-in-mathematics/
- Cairn Info. *Orientation scolaire et professionnelle des filles et des garçons au collège*, (2016) from https://www.cairn.info/revue-les-sciences-de-l-education-pour-l-ere-nouvelle-2016-1-page-91.htm
- Cambridge Dictionary. (n.d.). Guidebook. In *Cambridge Dictionary.org*. Retrieved April 6, 2023, from https://dictionary.cambridge.org/fr/dictionnaire/anglais/guidebook
- Cameron, J.M. (n.d.). Figure 16. Pexels. https://www.pexels.com/fr-fr/photo/internet-etre-assis-technologie-ipad-4145151/
- Cartwrigh, M. (2019, August 27). *Architecture in the Ancient World (Collection)—World History Encyclopedia*. https://www.worldhistory.org/collection/56/architecture-in-the-ancient-world/

- Cartwright, M. (2013, January 6). *Greek Architecture*. World History Encyclopedia. https://www.worldhistory.org/Greek_Architecture/
- Cartwright, M. (2018, March 14). *Roman Architecture*. World History Encyclopedia. https://www.worldhistory.org/Roman_Architecture/
- Chasses-au-trésor. *Chasse au trésor mathématique*, March 2018 from Chasse au trésor mathématique 2018 - Chasse au trésor (chasses-au-tresor.com)
- Climb Kalimnos. (n.d.). *Climb Kalimnos 3rd edition (2006).* https://climbkalymnos.com/new-guidebook-revisiting-the-history-of-kalymnos-guidebooks/
- Condroyer, C. (2022). Enquête IESF 2022 : bilan sur la situation des jeunes ingénieurs diplômés. *Studyrama*. https://grandes-ecoles.studyrama.com/ecoles-d-ingenieurs/actualites/enquete-iesf-2022-bilan-sur-la-situation-des-jeunes-ingenieurs-diplomes-10514.html
- Department for Education (2020). *Principles and practice: Motivationg and engaging students in further education maths*. https://www.et-foundation.co.uk/wp-content/uploads/2020/03/CfEM_Motivation-and-Engagement_Handbook.pdf
- Digital Unite (2022, July 13). *What is an EReader?*. https://www.digitalunite.com/hobbies-interests/reading-learning/what-ereader
- EURES (n.d.). *Labour market information : Italy*. https://eures.ec.europa.eu/living-and-working/labour-market-information/labour-market-information-italy_en
- European Dyslexia Association. *What is dyslexia*. (n.d.). Retrieved 21 March 2023, from https://eda-info.eu/what-is-dyslexia/
- Fédération Française des DYS. *Troubles DYS*. (2008, October 13). Retrieved 08 May 2023, from https://www.ffdys.com/troubles-dys
- Gaming for Skills (n.d.). *Game balancing theory*. https://www.gaming4skills.eu/wp-content/uploads/2022/01/Creator21_EN.pdf
- GDC. (2016, May 2). *'Magic: the Gathering': 20 Years, 20 Lessons Learned*. [Video]. YouTube. https://www.youtube.com/watch?v=QHHg99hwQGY
- GDC. (2021, December 8). Lessons from 'Duolingo': How to Make Learning Hard Things Easy. [Video]. YouTube. https://www.youtube.com/watch?v=i7_8TODHWRs&list=PL2e4mYbwSTbaXCp0QL8Pxr8PduktnXr4f&index=42

- Gill, E. (2020). *What is Your Teaching Style? 5 Effective Teaching Methods for Your Classroom*. https://resilienteducator.com/classroom-resources/5-types-of-classroom-teaching-styles/
- Grabowska, K. (n.d.). Figure 10. Pexels. https://www.pexels.com/fr-fr/photo/bureau-mathematiques-crayon-table-6958537/
- Herrity, J. (2023, March 17). *What Are Soft Skills? (Definition, Examples and Resume Tips)*. LinkedIN. https://www.indeed.com/career-advice/resumes-cover-letters/soft-skills
- IFTF 2023. *Careers of the future* (2023, February 13) from https://elabedu.eu/careers-of-the-future/
- Juicy_fish. (n.d.). Figure 13, "Game". FlatIcon. https://www.flaticon.com/fr/icone-gratuite/manette-de-jeu_1729449?term=game&page=1&position=7&origin=search&related_id=1729449
- Lavoué, E. (n.d.). *Impact de la ludification des séquences de cours sur la motivation*. https://www.reseau-canope.fr/agence-des-usages/impact-de-la-ludification-des-sequences-de-cours-sur-la-motivation.html
- Le Cam, M., Salles, F. (2020). *Note d'information n°20.47*. Direction de l'évaluation de la prospective et de la performance (DEPP). https://www.education.gouv.fr/timss-2019-mathematiques-au-niveau-de-la-classe-de-quatrieme-des-resultats-inquietants-en-france-307819
- Libération. *Les femmes ne s'intéressent pas aux sciences ? Vraiment ?*, (2021, February). https://www.liberation.fr/idees-et-debats/tribunes/les-femmes-ne-sinteressent-pas-aux-sciences-vraiment-20210210_ULZNLP7N25EWJLTLKKWXQKYANU/
- Lundin, J., Melkersson, J. (2022). *Gamification: Badges vs leaderboards as a motivational tool for university students learning a second language*. https://www.diva-portal.org/smash/get/diva2:1670386/FULLTEXT02
- M@ths en-vie. *Ancrer les mathématiques au réel !* from https://www.mathsenvie.fr/le-grand-rallye-mths-en-vie-pour-la-semaine-des-mathematiques-2021/

- MacTutor History of Mathematics Archive. (2000). *Mayan mathematics*. Retrieved 27 March 2023, from https://mathshistory.st-andrews.ac.uk/HistTopics/Mayan_mathematics/
- Manpower (2019). *45% des employeurs belges éprouvent des difficultés à remplir leurs postes vacants*. https://www.manpowergroup.be/fr/2020/01/29/45-des-employeurs-belges-eprouvent-des-difficultes-a-remplir-leurs-postes-vacants/
- MathCityMap. *Popular mathtrails* from https://mathcitymap.eu/en/
- McCray, B. (2013). *How to market a small town*. SmallBizSurvival. https://smallbizsurvival.com/2013/03/how-to-market-a-community.html
- Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science*. https://timssandpirls.bc.edu/timss2019/international-results/
- myCityHunt. *Jeux de piste numériques* from https://www.mycityhunt.fr/
- Nicholson, S. (2013). *Exploring Gamification Techniques for Classroom Management*. https://scottnicholson.com/pubs/gamificationtechniquesclassroom.pdf
- Objective Science International. *Voyages scientifique* from https://www.vacances-scientifiques.com/
- OECD (2017). Students' motivation to achieve. *PISA 2015 results (Volume III): Students' well-being*. https://www.oecd-ilibrary.org/docserver/9789264273856-9-en.pdf?expires=1680527650&id=id&accname=guest&checksum=E6220632A5316B9D2534966C682EB3A8
- OECD (2018). *PISA 2018 results: Combined executive summaries*. https://www.oecd.org/pisa/Combined_Executive_Summaries_PISA_2018.pdf
- OECD Pisa results. *Résumés*, (2018) from https://www.oecd.org/pisa/PISA2018%20_Resum%C3%A9s_I-II-III.pdf
- OECD Pisa results. *The difficulties of young people in the digital age*, (2019) from La dernière enquête PISA de l'OCDE met en lumière les difficultés des jeunes à l'ère du numérique - OCDE (oecd.org)
- Office de tourisme de Beaumont-de-Lomagne. *Les fers magiques de Bello Monte* from https://tourisme.malomagne.com/fr/fiche/equipement/jeu-de-piste-les-fers-magiques-de-bello-monte-beaumont-de-lomagne_TFO5846140/

- Parlock, J. (2021, December 7). *Magic The Gathering: What Are Top-Down And Bottom-Up Sets?*. Thegamer. https://www.thegamer.com/magic-the-gathering-top-down-bottom-up-sets/
- Pasaric, A. (n.d.). Figure 11. Pexels. https://www.pexels.com/fr-fr/photo/photo-de-batiments-pendant-la-nuit-2603464/
- Philippot, Y. (2019). *L'enfant qui regarde l'architecture, graines de citoyenneté semées dans les campagnes : expériences pédagogiques de l'architecture en Bretagne*. https://dumas.ccsd.cnrs.fr/dumas-02090368/document
- Ratmelia, B. (2019). *Review and summary: flow by Mihaly Csikszentmihalyi. NTU Library*. https://blogs.ntu.edu.sg/ntulibrary/2019/06/24/review-and-summary-flow-by-mihaly-csikszentmihalyi/
- Rossi, C. (2004). *Architecture and mathematics in ancient Egypt*. Cambridge University Press.
- S'CAPE. *Escape games pédagogiques* from https://scape.enepe.fr/
- Séjours éducatifs. *La ligue de l'enseignement* from https://www.sejours-educatifs.org/vv/voyage-scolaire/votre-recherche/
- TOM.travel - Monument Tracker. *L'application pour repérer les édifices alentours* from https://www.tom.travel/2016/03/09/monument-tracker-application-reperer-edifices-alentours/
- Université de Lille. *Rallye mathématiques des collèges* from https://rallye-irem.univ-lille.fr/
- Webzine IDELLO 2020. *Aimer les maths*, (2020, October) from https://webzine.idello.org/anxiete-mathematique-et-reussite-des-eleves-entretien-avec-le-professeur-daniel-ansari/
- WorldStrides. *Educational Travel & Educational Tours Abroad* from https://worldstrides.com/

References - Visuals

VISIT MATH

Co-funded by the European Union

Funded by the European Union. View and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Education and Culture Executive Agency (EACEA). Neither the European Union nor EACEA can be held responsible for them.

Project code: 2022-1-FR01-KA220-SCH-000090275

https://visitmath.eu/

FERMAT SCIENCE Une autre idée des maths

5th HIGH SCHOOL Agrinio · Greece

ISTITUTO STATALE "E. MONTALE" PONTEDERA

LogoPsyCom

ZS 31

YuzuPulse